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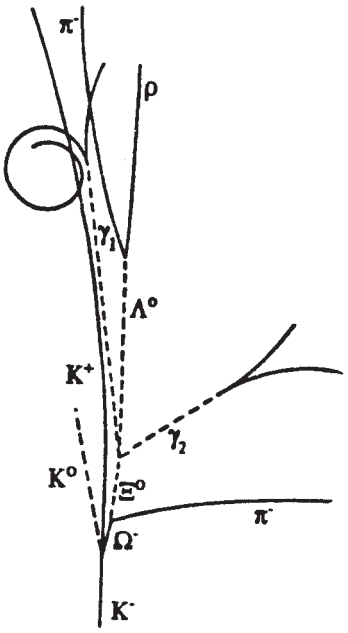

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WHAT'S WRONG WITH BLACK HOLE THERMODYNAMICS?

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Abstract

Not only is the Bekenstein expression for the entropy of a black hole a convex function of the energy, rather than being a concave function as it must be, it predicts a final equilibrium temperature given by the harmonic mean. This violates the third law, and the principle of maximum work. The property that means are monotonically increasing functions of their argument underscores the error of transferring from temperature means to means in the internal energy when the energy is not a monotonically increasing function of temperature. Whereas the former leads to an increase in entropy, the latter lead to a decrease in entropy thereby violating the second law. The internal energy cannot increase at a slower rate than the temperature itself.

Keywords: Black holes; Bekenstein entropy; Hawking temperature; Violation of thermodynamic laws; Mean values

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1 The 'Derivations' of the Bekenstein Entropy and the Hawking Temperature

The fallaciousness of the Bekenstein expression [?] for the entropy of a black hole is made apparent by the following derivation.

Consider the relativistic thermal wavelength,

$$\lambda = \frac{\hbar c}{kT}. \tag{1}$$

The only length that can be associated with a black hole is the Schwarzschild radius, so we set the thermal wavelength equal to it to obtain [?]

$$T = \frac{E_{\rm Pl}^2}{2kE}, \tag{2}$$

upon rearrangement, where $E_{\rm Pl} = \sqrt{(\hbar c^5/G)}$ is the Planck energy, and where the internal energy, E, has been confused with the rest energy, mc^2.

Hawking [?] claims that tiny black holes will radiate energy away at temperature eq:Hawking-T leading to complete evaporation in an explosion that would result in an intense burst of photons in the x-ray region.

Black bodies do not evaporate; in fact, Stefan's law is derived directly from the Clapeyron equation,

$$\frac{dp}{dT} = \frac{h}{T}, \tag{3}$$

where h is the enthalpy density, or the latent heat in this context.

The Clapeyron equation, eq:Clap, characterizes a first-order phase equilibrium. The analogy between thermal radiation and a solid-gas equilibrium, with the gas consisting of photons and the solid being the cavity walls which are both source and sink for photons, has long been known. [?]

If the cavity walls expand isothermally, photons will be injected into the cavity bringing with them a quantity of heat $Q = \sigma T^4 \Delta V$, where σ is the radiation constant and ΔV is the change in the volume of the cavity. The ratio, $Q/\Delta V = h$, is the latent heat of sublimation. The volume can be varied independently so as to keep the vapor pressure constant.

From the phase rule, $f = r - m + 2$, the number f of intensive parameters that can be varied independently is 1, if the number of components $r = 1$, and the number of phases $m = 2$. This is, indeed, verified: the chemical potential vanishes in the presence of an unconserved number of particles, and the pressure, $p = p(T)$, is the single intensive variable that can be varied independently. Under isobaric conditions there can be no change in the temperature, and, hence, C_p cannot be defined for black body radiation. [?] *Hence, black body radiation cannot be used as a mechanism of black hole evaporation.*

Appealing to the second law,

$$\frac{dS}{dE} = \frac{1}{T} = k\frac{2E}{E_{\rm Pl}^2}, \tag{4}$$

we obtain

$$S = k\frac{E^2}{E_{\rm Pl}^2} + \text{const.}, \tag{5}$$

upon integration. Expression eq:Bekenstein is to be considered as the fundamental relation for a black hole, and, since it does not contain any mechanical variables, no work processes are contemplated.

Differentiating eq:II a second time gives

$$\frac{d^2S}{dE^2} = -\frac{1}{T^2}\frac{dT}{dE} = -\frac{1}{T^2C} = \frac{2k}{E_{\rm Pl}^2},$$

so that the heat capacity, C is

$$C = -\frac{E_{\rm Pl}^2}{2kT^2}. \tag{6}$$

The fact that $C < 0$, we are told, should not dissuade from considering eq:Bekenstein as the true entropy of a black hole because the heat capacities of many stars are negative.

2 Can Heat Capacities be Negative?

However, those heat capacities have nothing to do with the second law. They refer to *polytropes*, which are defined by an equation of state of the

form:

$$pV^n = \text{const.}$$

If during the process of stirring, a quantity of heat, dQ, is supplied then it will be proportional to the change in temperature, dt, viz.,

$$dQ = cdt, \tag{7}$$

where t is the empirical temperature. Expression eq:C-bis is the definition of a polytropic change. [?] If the constant, c, is zero, the change is adiabatic, while if it is infinite, the change is isothermal. It was this 'heat capacity' that was thought can become negative. [?] Even if this were true, it has nothing to do with the second law.

Consider an ideal gas: $pV/t = A := C_p - C_V$, where C_p and C_V are the heat capacities at constant pressure and volume, respectively. The doctrine of latent and specific heats demands [?]

$$dQ = C_V dt + L_V dV \tag{8}$$

if the pair of independent variables are (t, V), and L_V is the latent heat. Introducing eq:C-bis and dividing through by t results in

$$\frac{dt}{t} + (n-1)\frac{dV}{V} = 0, \tag{9}$$

since $L_V = p$ for an ideal gas, and where the polytropic index n is defined as [?, p. 41]:

$$n = \frac{C_p - c}{C_V - c}. \tag{10}$$

Integrating eq:adiabat gives

$$V^{n-1}t = \text{const.} \tag{11}$$

Thus, dQ =$C_V dt + pdV = C_V dt - \frac{pV}{t(n-1)}dt$
$= \left[C_V - \frac{A}{n-1}\right] dt$ so that

$$c = \frac{C_V(n-1) - A}{n-1} \tag{12}$$

can become negative in the interval [?]

$$1 < n < \gamma, \tag{13}$$

where $\gamma := C_p/C_V$.

However, it is important to bear in mind:

- temperature and volume are no longer independent quantities, but, rather are related to the adiabatic relation, eq:z, and
- this has nothing to do with the second law.

It is the concavity of the entropy which is the essence of the second law [?], and which the Bekenstein entropy, eq:Bekenstein, violates. We are now going to show that for an ideal, monatomic, gas the heat capacity eq:c-bis always vanishes because the second inequality in eq:ineq is actually an equality.

The polytropic index, n, is related to the Grüneisen parameter, s, defined by the equation of state [?, p. 40],

$$s = pV/E. \tag{14}$$

For the ideal gas under consideration,

$$s = \gamma - 1 = . \tag{15}$$

The relation between s and n follows from the fact that V^s is the integrating factor for [?, p. 42]

$$dQ = dE + pdV = dE + s\frac{E}{V}dV.$$

Upon multiplication on both sides by V^s there results the perfect differential,

$$V^s dQ = d\left(EV^s\right),$$

implying that tV^s = const. is the adiabatic condition for an ideal gas. Comparing this with eq:z we conclude that $s = n - 1$, and $n > 1$. According to eq:s, $n = \gamma$, and from its definition, eq:n, we conclude that c vanishes. *Thus, for an ideal gas, c can never be negative.*

3 Proof of the Incorrectness of the Bekenstein-Hawking Expressions

We now divide the black hole up into number of cells, with masses m_j, and with specific heats, $c(T_j)$. Initially, they are separated by adiabatic impermeable walls which are subsequently replaced by diathermal walls. We want to compare greater and less constrained states of equilibrium, and, thereby, determine the final common temperature. [?]

The heat capacity eq:C, derived from the Bekenstein entropy, eq:Bekenstein, would lead to a conservation of energy,

$$\Delta E = -c\sum_j m_j \int_{T_j}^{T_f} \frac{dT_j}{T_j^2} = c\left\{\frac{M}{T_f} - \sum_j \frac{m_j}{T_j}\right\} = 0,$$

from which we would deduce the final equilibrium temperature as the weighted *harmonic* mean temperature,

$$\frac{1}{T_f} = \frac{1}{M}\sum_j \frac{m_j}{T_j}, \tag{16}$$

where

$$M = \sum_j m_j,$$

is the total mass. However, eq:harmonic would violate the third law of thermodynamics, which, in its strong form reads

$$\lim_{T\to 0} S = 0. \tag{17}$$

It is, therefore, necessary to assume that the T_j are nonnegative which implies that all means of order less than zero be taken as zero. [?]

For simplicity consider two subsystems. Then multiplying eq:harmonic through by cM gives

$$E_f = \frac{cM}{T_f} = (E_1 + E_2) = \frac{cm_1}{T_1} + \frac{cm_2}{T_2}, \tag{18}$$

which says simply that energy is conserved. Likewise, the second law reads

$$\Delta S = c\left\{\frac{M}{T_f^2} - \left(\frac{m_1}{T_1^2} + \frac{m_2}{T_2^2}\right)\right\} \geq 0,$$

which, in terms of the energies, implies

$$\frac{E_f^2}{M} \geq \frac{E_1^2}{m_1} + \frac{E_2^2}{m_2}. \tag{19}$$

Again, for simplicity, suppose $m_1 = m_2 = m$ so that $M = 2m$, and eq:wrong reduces to

$$(E_1 + E_2)^2 \geq 2\left(E_1^2 + E_2^2\right).$$

This simplifies to

$$2E_1E_2 \geq E_1^2 + E_2^2,$$

which is blatantly wrong.

If we call M_p, the mean of order p, then because it is a monotonic increasing function of p [?],

$$M_{p+1} \geq M_p, \qquad \forall -\infty < p < +\infty. \tag{20}$$

But because E is *not* a monotonically increasing function of T, we cannot use eq:MM to guarantee that the means of the energy will behave in the same way as the temperature. This is an internal consistency condition of thermodynamics meaning that it outlaws means of order $p < 0$.

Thus, it is not like what has been claimed: "heat capacities proportional to negative powers of T can well be considered except at very low T." [?] Thermodynamics is self-consistent, and violating any one of its laws leads to internal contradictions in the others.

If eq:harmonic were a viable, common final temperature, it would violate the principle of maximum work which identifies the geometric mean as the final common temperature. This occurs for the ideal gas in which the transition from initial to final states is *reversible*, [?] $\Delta S = c\sum_j m_j \int_{T_j}^{T_f} \frac{dT_j}{T_j}$

$= c\left\{M \ln T_f - \sum_j m_j \ln T_j\right\} = 0$, leading to the weighted geometric mean temperature,

$$T_f = \left(\prod_j T_j^{m_j}\right)^{1/M}, \tag{21}$$

as the final mean temperature. The maximum work would then follow from the first law, W=-$\Delta E = -c\sum_j m_j \int_{T_j}^{T_f} dT_j$
$= c\left\{\sum_j m_j T_j - \left(\prod_j T_j^{m_j}\right)^{1/M}\right\} > 0$, where the inequality follows directly from the arithmetic-geometric mean inequality.

This shows that the internal energy cannot increase more slowly than the temperature itself, and, consequently, rules out thermal equations of state of the form eq:Hawking-T. This is entirely reasonable since kT is the average kinetic energy with which the particles in a gas move about. The internal energy may increase faster than the temperature due to creation and annihilation of particles, such as in a photon gas, but it cannot increase at a slower rate.

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TWISTED TALES FROM STRING THEORY

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Abstract

The Regge trajectories, upon which string theory is based, behave as rigid rotators rather than vibrating strings. The same relation, between the angular momentum, and the square of the mass, can be found in gravity, the electroweak, and strong interactions. The angle deficit for cosmic strings is shown to be an angle excess that is related to the increase of the circumference of a uniformly rotating disc. Schrödinger's time independent equation with a centrifugal barrier gives an automorphic function that can be constructed as the ratio of its two independent solutions for values of the angular momentum lying outside of their positive, integer values. If the fixed points 0 and ∞ in the z-plane correspond to $-i$ and $+i$ in the ω-plane, then elliptic substitutions tessellate the ω-plane in the form of cresents, while if the fixed points correspond to -1 and $+1$ in the ω-plane are source and sink, just like the lines of force between positive and negative charges. The unit disc undergoes a stretching by a Lorentz transform. With the inclusion of a third fixed point, the Schwarz-Christoffel mapping of the z-plane onto the triangle functions of Schwarz is the incomplete Euler beta function found by Veneziano for the 4-point scattering amplitude.

Keywords: Regge trajectories; String theory; Angular momentum-mass relation; Automorphic functions; Schwarz-Christoffel function as a scattering amplitude

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1 Flexible Strings or Rigid Rotators?

It is well-known that divergences in physical theories occur because of the approximation of treating particles as idealized points. Once their finite extension is accounted for, these divergences should be tamed. String theory compliments itself as being "the first quantum theory, ever, to include gravity without also including ultraviolet divergences." [?] However, its nonunitary nature has led to inconsistencies relating to the breakdown of conservation laws, probability, and causality. [?]

Heisenberg realized that there were two types of 'fundamental' constants: $\hbar$ and c, and e and G. The latter pair contain the intrinsic coupling strengths to the electrodynamic and gravitational fields, the charge, e, and the Newtonian gravitational constant, G. The first pair of constants determine the nature of the interaction; when Planck's constant, $\hbar$, appears, quantum effects are important, and when the speed of light appears c, relativistic effects occur. [?] Now, a fundamental length, ℓ, should be related to a fundamental mass, m, through the Compton relation, $\ell = \hbar/cm$, which is the linear dimension for constructing the best-localized state from a wave packet containing only plane-wave components. That such a distance cannot be determined from e, $\hbar$ and c is intended to mean that electromagnetic interactions are carried by particles with no mass. We may say there is no effective range of interaction for electromagnetic waves, or that the range is infinite. Not so with gravity for a length can be determined from the combination $\sqrt{(\hbar G/c^3)}$, and, consequently, a mass, $\sqrt{[(\hbar c)/G]}$. This is reason enough to believe that gravity waves, if they exist, should not behave as electromagnetic waves and travel at speed c, even in the linear approximation to the gravitational equations of general relativity. Moreover, the purported carriers of the gravitational interaction, gravitons, should likewise be limited in extent in contrast to weightless photons.

The reason why there are no divergences in string theory is because it introduces a single parameter, ε, which is the linear energy density. It is related to the string length ℓ_S by

$$\ell_S = \hbar c\sqrt{\alpha'} = \sqrt{\left(\frac{\hbar c}{2\pi\varepsilon}\right)}, \tag{1}$$

where

$$\alpha' = \frac{1}{2\pi\varepsilon\hbar c}. \tag{2}$$

The constant eq:alpha has been singled out because it is the slope of a Regge trajectory. We will show that ε is analogous to G^{-1}, and to a whole slew of other coupling strengths. However, the association of an energy density with a string tension will be seen to be inappropriate.

Consider a *rigid* rod of length $2r_0$, as shown in Fig. ??. It is rather weird that this will become the model of a *flexible* string, which is envisioned as connecting quark pairs, $q\bar{q}$, shown in Fig. ??. The rigid bar is free to rotate about its center of mass, and the velocity of its ends is c. Since we are considering a *rigid* rotation, $r_0 = c/\omega$, which defines the angular velocity ω; for any other velocity, $r\omega < c$.

[width=0.3]bar.jpeg

Figure 1: A rigidly rotating bar used to calculate the relation between the angular momentum and the mass of a hadron. The ends of the bar travel at the speed of light.

The total string energy will be given by [?]

$$E_S = 2\int_0^{r_0} \frac{\varepsilon}{\sqrt{(1 - v^2/c^2)}} dr = 2\int_0^{c/\omega} \frac{\varepsilon}{\sqrt{(1 - r^2\omega^2/c^2)}} dr. \tag{3}$$

Now introducing a change of variable $r = (c/\omega)\sin\vartheta$ in eq:E results in

$$E_S = \frac{2c\varepsilon}{\omega}\int_0^{\pi/2} d\vartheta = \pi\varepsilon\frac{c}{\omega}. \tag{4}$$

[width=0.5]string.jpeg

Figure 2: The analogy between the lines of force (left) between two opposite charges and the color lines of force between quark pairs (right). The lines are pictorially seen to hold together the quarks by a tube or string on account of the strong self-interaction between gluons, the carriers of the color field.

The total angular momentum, J, in units of $\hbar$, can also be calculated relativistically, according to its definition

$$J_S = \frac{2}{\hbar c^2}\int_0^{r_0} \frac{\varepsilon r v}{\sqrt{(1-v^2/c^2)}} dr = \frac{2\varepsilon}{\hbar c^2}\int_0^{c/\omega} \frac{r^2\omega dr}{\sqrt{(1-r^2\omega^2/c^2)}} \tag{5}$$

Again, introducing the change of variable into eq:J results in

$$J_S = \frac{2\varepsilon c}{\hbar\omega^2}\int_0^{\pi/2} \sin^2\vartheta \; d\vartheta = \frac{\pi}{2}\frac{\varepsilon c^2}{\hbar\omega^2}. \tag{6}$$

Now, eq:E-bis and eq:J-bis must be valid for whatever angular velocity we may choose. Eliminating ω from the two equations gives

$$J_S = \alpha' E_S^2, \tag{7}$$

where the constant of proportionality is given by eq:alpha. The Regge trajectory, eq:Regge has zero intercept. A non-vanishing intercept cannot be derived from classical phenomenologically. Since $v \propto r$, the rotation is rigid, and hardly resembles a flexible string. But, it does resemble other theories where there is but a single parameter that fixes the scale.

From eq:E-bis, the string mass may be simply written down as

$$M_S = \hbar/c\sqrt{(\hbar c/\varepsilon)} = \sqrt{\left(\frac{\varepsilon\hbar}{c^3}\right)}, \tag{8}$$

where we drop factors of 2π since we are interested only in the ratios of fundamental constants. In terms of eq:MS, the Regge trajectory, eq:Regge can be written as

$$J_S = \frac{M^2}{M_S^2} = \frac{G_S(Mc^2)^2}{\hbar c}, \tag{9}$$

where the 'gravitational' coupling $G_S = 1/\varepsilon$.

The characteristic frequency of the string,

$$\omega_S = c/\ell_S = \sqrt{\left(\frac{\varepsilon c}{\hbar}\right)}, \tag{10}$$

is not the characteristic frequency of a real string [?, p. 75],

$$\omega_S r = \sqrt{\left(\frac{\varepsilon}{\varrho}\right)} \tag{11}$$

where ϱ is the linear mass density, and ε is the string tension [cf. eq:string-bis below].

It is already apparent that the so-called Planck scale inverts mass and length so that the mass and length are $\mathrm{M}_G = \sqrt{\left(\frac{\hbar c}{G}\right)}$, $\ell_G = \sqrt{\left(\frac{G\hbar}{c^3}\right)}$. It is apparent from eq:MG that we can write the Regge trajectory as

$$J_G = \frac{M^2}{M_G^2} = \frac{GM^2}{\hbar c}, \tag{12}$$

which is a dimensionless ratio, just like the fine structure constant,

$$J_\gamma = \frac{e^2}{\hbar c}. \tag{13}$$

If eq:FS is truly angular momentum, then light has a rotational inertia of

$$\mathcal{I} = \frac{\infty \quad \zeta}{\infty \ni \imath \omega},$$

so that its angular momentum is

$$L = J_\gamma \hbar = \mathcal{I}\omega = \rceil^{\in}/\rfloor.$$

If eq:ReggeG is to hold on all scales it must hold on the Planck scale, which will serve as a check of self-consistency. Writing eq:ReggeG in the form

$$\ell_G^2 \omega = \frac{GM_G}{c} \tag{14}$$

allows us to solve for the characteristic angular velocity. We thus find

$$\omega_G = \sqrt{\left(\frac{c^5}{G\hbar}\right)}. \tag{15}$$

This could have been obtained directly from eq:ellG, i.e., $\omega_G = c/\ell_G$. Finally, the rotational inertia is

$$\mathcal{I}_\mathcal{G} = \frac{\zeta}{\rfloor} \ell_\mathcal{G}. \tag{16}$$

Analogous considerations apply to the weak interaction of β-decay, that was first predicted by Fermi back in 1934. The Regge trajectory for β-decay is

$$J_F = C\frac{M^2}{M_P^2} = \frac{G_F(Mc/\hbar)^2}{\hbar c}, \tag{17}$$

where M_P is the proton mass, $C = 10^{-6}$, a numerical constant that must be obtained from experiment, and

$$G_F = \hbar c\left(\frac{10^{-3}\hbar}{M_P c}\right)^2, \tag{18}$$

is the coupling strength of the four-fermion contact interaction. G_F has the dimensions of energy$\times$volume, and the last factor in eq:ReggeF is the square of the inverse of the Compton wavelength for a particle of mass M. It is what is required to make J_F dimensionless.

The effective range of the weak interaction is

$$\ell_F = \sqrt{\left(\frac{G_F}{\hbar c}\right)}, \tag{19}$$

which is the inverse of the string length, eq:ellS. Consequently, the mass of the weak interaction is

$$M_F = \hbar\Big/\sqrt{\left(\frac{G_F c}{\hbar}\right)} = \sqrt{\left(\frac{\hbar^3}{G_F c}\right)} = 10^3 M_P. \tag{20}$$

Given eq:ellF, the characteristic angular velocity is found to be

$$\omega_F = \sqrt{\left(\frac{\hbar c^3}{G_F}\right)}. \tag{21}$$

If we didn't know eq:MF, we could calculate it from the Regge trajectory eq:ReggeF by introducing the definition of angular momentum, $J_F = M_F\ell_F^2\omega_F/\hbar$, where eq:ellF and eq:omegaF are introduced. We would then find eq:MF, which is not the mass of the $W^{\pm}$ bosons. Why?

The interconversion of neutrons into protons are charge-changing reactions, and as such involve the transfer of charged $W^{\pm}$. These 'charged current' reactions are in contrast to electromagnetic scattering where the

exchanged particle, the photon, is uncharged. Such scattering is a 'neutral' current reaction. But, if the two are to be of the same strength, then G_F, which has the dimensions of $e^2 \times L^2$, must be of the form

$$G_F = g^2 \lambda_W^2, \tag{22}$$

where λ_W is the Compton wavelength of the $W^{\pm}$ bosons. Setting eq:GF equal to eq:GF-bis we find

$$g^2/\hbar c = \left(10^{-3}\frac{M_W}{M_P}\right)^2, \tag{23}$$

which characterizes the coupling of the $W^{\pm}$ to the fermions, whose masses are $M_W = 80\text{GeV}/c^2$.

Introducing eq:GF-bis into eq:ReggeF gives

$$J_F = \frac{g^2}{\hbar c}\left(\frac{M}{M_W}\right)^2; \tag{24}$$

which is analogous to the pion-nucleon coupling constant,

$$f^2/\hbar c = (g^2/\hbar c)(M_\pi/M_P)^2,$$

where $f^2/4\pi$ is the pion-nucleon coupling constant of the strong interaction, having dimensions of the fine structure constant. [?]

Introducing eq:GF-bis into the Fermi mass, eq:MF, results in

$$M_F = \sqrt{\left(\frac{\hbar c}{g^2}\right)} M_W = 10^3 M_P. \tag{25}$$

If we assume, along with Salam and Weinberg, that electromagnetic and weak interactions have the same intrinsic coupling of bosons to leptons, i.e. $g \simeq e$, thereby unifying the 'electroweak interaction,' we find the mass of the $W^{\pm}$ bosons as

$$M_W = 10^3 M_P \sqrt{J_\gamma} = 0.085 \times 938 \times 10^3\text{MeV} = 80\text{GeV}. \tag{26}$$

The rotational inertia of the weak interaction is

$$\mathcal{I}_{\mathcal{F}} = \frac{\ell^{\in}}{\mathcal{M}_{\mathcal{F}}\rfloor^{\in}} = \mathcal{M}_{\mathcal{F}}\lambda_{\mathcal{F}}^{\in}. \tag{27}$$

10mm—|—|—|—|—|—|— *name* M_X ℓ_X ω_X $\mathcal{I}_\mathcal{X}$ J_X

string $X = S$ $\sqrt{\left(\frac{\varepsilon\hbar}{c^3}\right)}$ $\sqrt{\left(\frac{\hbar c}{\varepsilon}\right)}$ $\sqrt{\left(\frac{\varepsilon c}{\hbar}\right)}$ $\frac{\hbar}{\omega_S}$ M^2/M_S^2

gravity $X = G$ $\sqrt{\left(\frac{\hbar c}{G}\right)}$ $\sqrt{\left(\frac{G\hbar}{c^3}\right)}$ $\sqrt{\left(\frac{c^5}{G\hbar}\right)}$ $\frac{\hbar}{\omega_G}$ M^2/M_G^2

weak interaction $X = F$ $\sqrt{\left(\frac{\hbar^3}{G_F c}\right)}$ $\sqrt{\left(\frac{G_F}{\hbar c}\right)}$ $\sqrt{\left(\frac{\hbar c^3}{G_F}\right)}$ $\frac{\hbar}{\omega_F}$ M^2/M_F^2

This identifies the mass in eq:ReggeF-bis as eq:MF-bis.

Consequently, the relation $J \sim M^2$ appears to be ubiquitous in particle physics and has nothing to do with the behavior of a flexible string. The factors are shown in the Table [1] The only place we expect an 'intermediate particle' is in the weak interaction due to the presence of a numerical factor in the last column. The introduction of the fine structure constant, eq:g, in analogy with the exchange of a neutral boson between leptons will then reduce M_F to $\sqrt{J_\gamma} M_F = M_Z$ since the orders of magnitude of $W^\pm$ and Z^0 are the same.

2 Dynamic Phenomena Related to $J \sim M^2$

The last column of the Table establishes a dynamic equilibrium between the repulsive centrifugal force and attractive forces. The best known is that of gravity, where the relativistic virial is

$$\frac{v^2}{c^2} = \frac{2GM}{c^2\lambda} = \frac{\alpha}{\lambda}, \tag{28}$$

with $\lambda = c/\omega$, and, α is the Schwarzschild radius, $2GM/c^2$. Since we are considering the volume per particle, we can interchange the radius, r, and the wavelength, λ. Then dividing through by r in eq:gravity it says that the centrifugal forces just balance the attractive force by a central mass M due to gravity. If, instead of the mass, the density, $\varrho = M/r^3$, is constant, eq:gravity reduces to

$$\omega_{\text{rot}} = v/\lambda = 1/\sqrt{(G\varrho)}. \tag{29}$$

[1] Albeit the electroweak and gravitational interactions both show $J \sim M^2$ behavior, as a result of their fine structure, $g^2/\hbar c$ and $GM^2/\hbar c$, there is a vast difference in their canonical versus non-unitary descriptions. [?]

This is the maximum frequency that a star can rotate without its matter being torn off its surface by the centrifugal force. [?]

According to the last column for the weak interaction in the Table, the mass of the intermediate particle would be given by

$$M_W = 10^3 M_P \frac{v}{c}. \tag{30}$$

Comparing eq:MW with eq:weak we find

$$\frac{v^2}{c^2} = J_\gamma \ll 1, \tag{31}$$

and which indicates that the electroweak interaction is a non-relativistic effect. Although β-decay can be relativistic, eq:weak-bis says that the relation $J \sim M^2$ holds only in the non-relativistic regime. It is well-known that Regge poles characterize 'soft-scattering' processes where the created particles carry away small momenta in the direction normal to the beam. Since Regge theory has little to do with 'hard-scattering' processes, we would not expect such trajectories exist for such processes. However, eq:MF implies another mass, which is 11.7 times that of M_W. For such a particle we would, again, expect $J \sim M^2$ to hold.

Finally, for the 'string,' we have

$$\frac{v^2}{c^2} = \frac{M}{M_S}. \tag{32}$$

If the string has constant density, $\varrho = M/\lambda$, eq:string becomes

$$\frac{v}{c} = \sqrt{\left(\frac{\varrho c^2}{\varepsilon}\right)} = \frac{r}{r_0}. \tag{33}$$

In general, at intermediate points along the string, $\varepsilon > \varrho c^2$, and only at the ends will there be an equality. In fact, eq:omegaS-bis would predict superluminal velocities independent of where one is on the string. The expression for the mass of a string, eq:MS, shows that the higher the energy density, the larger the mass. The Regge trajectory, eq:Regge, shows that the higher the energy density, the smaller will be its angular momentum. This is in glaring contrast to the transverse vibrations of a string where the higher the tension, or the lighter the string, the larger will be the velocity of propagation. [?, p. 75]

3 Cosmic Strings are Out of this World

Because a string has mass, one would be inclined to believe that there will be gravitational attraction. This is what Newtonian theory tells us, but not Einstein's relativistic theory for the tension of a string would exert a 'negative' gravitational attraction that would cancel precisely the positive gravitational attraction of its mass. Yet, while strings exert no gravitational attraction, they will affect the geometry of planes perpendicular to the string. [?]

Suppose we revolve around a string at a fixed distance r. Surprisingly, we would find that the circumference of our circle is not $2\pi r$, but [?]

$$C = (2\pi - \Delta)\, r, \tag{34}$$

where Δ is the 'deficit angle.' According to general relativity it is

$$\Delta = \frac{8\pi G\varepsilon}{c^4} = \frac{8\pi G\varrho}{c^2}. \tag{35}$$

Expression eq:Delta looks odd because instead of having one parameter measuring the strength of the coupling we now have two.

If the cosmic string is placed between a source of light and an observer, it can produce gravitational lensing, where light avails itself of more than one geodesic to reach the observer. Such compact objects can thus produce more than two images, and since the geometry is curved, the images may appear distorted, and could be quite different. This, according to string theorists, would provide firm evidence for the existence of cosmic strings.

Apart, from our earlier objection to eq:Delta, eq:circ can't be right. According to Fermat's principle of least time, the optical path,

$$I = c\tau = \int \eta\sqrt{(2T)}\; dt, \tag{36}$$

connecting any two arbitrary points, should be stationary. T is the kinetic energy per unit mass, and it is related to the Euclidean metric according to

$$\sqrt{(2T)}\; dt = ds = \sqrt{\left(dr^2 + r^2 d\varphi^2\right)}, \tag{37}$$

in polar coordinates.

Rather, if the medium is inhomogeneous, the index of refraction, η, will vary with r, but not with φ. If light is propagating in a gravitational field, the optical path length is [?]

$$I = \int \eta ds = \int ds' = \int \sqrt{\left(1 + \frac{2\alpha}{r}\right)}\ ds. \tag{38}$$

Moreover, if we consider $r = \text{const.}$, then eq:Fermat-bis will reduce to

$$C = \int_0^{2\pi} ds' = r \int_0^{2\pi} \sqrt{\left(1 + \frac{2\alpha}{r}\right)}\ d\varphi = \sqrt{\left(1 + \frac{2\alpha}{r}\right)}\, 2\pi r. \tag{39}$$

For weak fields, eq:circ-bis would be approximated by

$$C = \left(1 + \frac{2GM}{r}\right) 2\pi r. \tag{40}$$

So, instead of having an angle 'deficit', we have an angle 'excess.' This is none other than Einstein's old result of the circumference of a uniformly rotating disc [?]. Einstein reasoned, incorrectly, that the rulers lined up along the circumference of the disc would undergo a FitzGerald-Lorentz contraction, so more would be needed when the disc is in motion than when it was at rest. This is fallacious since the circumference too would likewise undergo the same contraction. Expression eq:circ-tris would also allow Einstein to conclude that a gravitational field mimics uniform acceleration so that all forms of acceleration are equivalent. This he called his 'equivalence principle.' This also is inaccurate since the gravitational field appears as a cause for inhomogeneity in the expression for the index of refraction, eq:Fermat-bis, and, in its absence, the disc would still rotate with unform acceleration.

If the latter is true then we would expect that the circumference is still $2\pi r$. That would be if we are in Euclidean space. Rather, in hyperbolic space the metric is not given by eq:KE, but, rather by [?, p. 487]

$$d\bar{s}^2 = d\bar{r}^2 + R^2 \sinh^2(\bar{r}/R) d\varphi^2, \tag{41}$$

where $\bar{r} = R \tanh^{-1} r/R$ is the straight-line segment in hyperbolic space, and R is the radius of curvature. A common, and almost universal, error is to set $r = R \sinh \bar{r}/R$. [?] While it is true that the surface area is

$4\pi R^2 \sinh^2 \bar{r}/R$, it is so for a different reason, i.e., the effect of distortion has not been taken into account. [?] This is related to the fact that the sum of the angles of a triangle is inferior to π.

At constant r, the Beltrami metric, eq:Beltrami gives a circumference

$$C = \int_0^{2\pi} d\bar{s} = \frac{r\, d\varphi}{\sqrt{(1 - r^2\omega^2/c^2)}} = \frac{2\pi r}{\sqrt{(1 - r^2\omega^2/c^2)}}, \tag{42}$$

which is Einstein's old result [?], where we have set the radius of curvature, $R = c/\omega$. As the derivation shows, it has been obtained from the wrong reasons. [?] Thus, all forms of acceleration are not equivalent. The centrifugal potential is already included in the Euclidean metric, $\dot{r}^2 + L^2/r^2$, where $L = r^2\dot{\varphi}$ is the angular momentum per unit mass. Hyperbolic space modifies the result $L =$ const. to read

$$\frac{L^2}{r^2\,(1 - r^2\omega^2/R^2)} = \text{const.} \tag{43}$$

The same result is obtained in the general theory of relativity [?, p. 349 formula (18)], provoking the comment that eq:L "cannot in general be interpreted as angular momentum, since the notion of a 'radial vector' occurring in the definition of the angular momentum has an unambiguous meaning in a Euclidean space."We strongly disagree with Møller. The effect here is the destruction of the uniformity of space by a static gravitational field, and has the same effect in varying the index of refraction in air which decreases with altitude.

Cosmic strings are supposedly left-overs from the early universe, and can stretch across the observable universe [?]. If they are to be detected by gravitational lensing then string theorists should be looking for angular excess instead of deficit.

4 Black Holes in Anti-de Sitter Universes of 5 Dimensions

String theory needs 'supersymmetry' to introduce fermions which also lowers the dimensions from 26 to 10. Then starting with 10 dimensional

super string theory and compactifying six of those dimensions will result in a 4 dimensional super symmetric universe. However, this will not cover the case of black holes which supposedly require 5 dimensions in an anti-de Sitter (AdS) universe. This is indeed intriguing for it not only reproduces results which are blatantly incorrect, it needs 5 dimensions to do so! Black holes supposedly radiate thermal energy with a black body spectrum [?]. String theorists claim that 5 dimensions are required, but consider black body radiation in only 3 dimensions!

The good news, string theorists tell us, is that AdS supports black holes in 5 dimensions. The Schwarzschild metric [?, p. 325 formula (79)]

$$ds^2 = -\left(1 - \frac{\alpha}{r} - \lambda r^2/3\right) dt^2 + \left(1 - \frac{\alpha}{r} - \lambda r^2/3\right)^{-1} dr^2 + r^2 \left(\sin^2 \vartheta d\varphi + d\vartheta^2\right), \tag{44}$$

where λ is a small, positive, constant, gets mutilated into [?, p. 554 formula (23.104)]

$$ds^2 = -\left(1 + \frac{r^2}{R^2} - \frac{r_0^2}{r^2}\right) dt^2 + \left(1 + \frac{r^2}{R^2} - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2, \tag{45}$$

by the string theorists. "The parameter r_0 tells us that we have a black hole" in an AdS of 5 dimensions, where $d\Omega_3^2$ "denotes the metric of a 3-sphere S^3 of unit radius." Moreover, "one can write r_0 in terms of the mass M of the hole and the 5 dimensional Newton's constant: $r_0^2 \sim G^{(5)}M$." [?, p. 555]

String theorists also tell us that the Schwarzschild radius, r_+ is that value of r that nullifies the coefficient of dt^2 in the metric eq:Schwarz-bis. It will only be proportional to r_0 in the limit $r \ll R$. In general,

$$\frac{r_+}{r_0} = \frac{1}{1 + r_+^2/R^2} < 1,$$

which makes the dubious claim that the Schwarzschild radius is always less than r_0, which is proportional to the mass M of the black hole. Moreover, R, the radius of curvature, is supposedly not a limit to the size of a black hole: Small black holes are those for which $r_+ \ll R$, while large black holes have $r_+ \gg R$.

[width=0.5]BZ.jpeg

Figure 3: This figure is taken from [?] which plots the product of the Hawking temperature and the AdS radius versus the Schwarzschild radius r_+ scaled by the same radius. For each value of the Hawking temperature above T_0 there exist two black holes. For temperatures less than T_0 there cannot be "a black hole dual since there is no black hole"

Now enters thermodynamics: For any metric of the form

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + \cdots,$$

the Hawking temperature is defined by, $T = f'(r_+)/4\pi$ [?, p. 556], where the prime stands for the differentiation with respect to r,

$$T_H = \frac{R^2 + 2r_+^2}{2\pi r_+ R^2}, \tag{46}$$

which has been evaluated at $r = r+$. Consequently, small black holes will have a Hawking temperature of

$$T = \frac{1}{2\pi r_+}, \tag{47}$$

while for large black holes,

$$T = \frac{r_+}{\pi R^2}. \tag{48}$$

String theorists even readily admit that "This is unusual: once a black hole is large enough its temperature grows with its size." [?, p. 556] Nonetheless, "[t]his is an important qualitative feature of black holes."

For a fixed radius of curvature, R, we are told that the Hawking temperature has a lower bound given by

$$T_H \geq T_0 = \frac{\sqrt{2}}{\pi R},$$

where the lower bound, T_0, is attained at $r_+/R = 1/\sqrt{2}$. This is depicted in Fig. ??. At temperature T_1 there occurs a "Hawking-Page transition."

Temperature may mean different things to different people. But, all would agree that temperature is a *monotonic* function of whatever it is a

function of. If the universe is expanding into a void, then the adiabatic expansion should show a continual decrease in temperature as the radius of the sphere increases, $TV^{1/3} = \text{const}$. We certainly should not expect something like eq:piccolo and eq:grande occurring in the same system under the same conditions.

Nonetheless, it is absolutely necessary in order to show that black holes radiate with a black body spectrum. The last term in the mutilated metric, eq:Schwarz-bis shows that the horizon is a three-sphere, S^3, of radius r_+, although previously we were told that it was for a unit radius. The metric, eq:Schwarz-bis, for fixed r becomes

$$ds^2 \simeq \frac{r^2}{R^2}\left[-dt^2 + R^2 d\Omega_3^2\right]. \tag{49}$$

The following conclusions are *non-seguiturs* [?, pp. 555-556]:

- "While the radius, R, is immaterial in the zero-temperature case, it is not immaterial now."
- "The radius R is the radius of the sphere where the field theory at temperature T_H lives."
- "One cannot rescale with impunity the time coordinate t in eq:BZ because at non-zero temperature the time coordinate carries information about temperature."
- "The product RT_H of the radius and the temperature is the *only* invariant information."

A generalization of the Bekenstein expression for the black hole entropy says that the black hole entropy is the ratio of the 'area' of the event horizon divided by the 5-dimensional Newtonian constant, $G^{(5)}$:

$$S_{BH} = \frac{A_{\text{hor}}}{4G^{(5)}}. \tag{50}$$

Now comes the punchline: The last term in the metric eq:Schwarz-bis shows that the horizon is a 3-sphere of radius r_+ so the area is $A_{\text{hor}} = 2\pi^2 r_+^3$, and not proportional to the square of the radius as it would be had

the field theory lived in real 3-dimensional space. Then from eq:grande we see that r_+ is proportional to the temperature T_H so that the cube of one will give the cube of the other. And since we *know* that the entropy of black body radiation is proportional to the cube of the temperature, it would seemingly make compatible the entropy of a black hole with the entropy of black radiation.

The whole thing is a *mise en scène*, and even if we didn't know what the gravitational constant looks like in 5-dimensions we could find it by comparing eq:S with the entropy of black body radiation. This would give $G^{(5)} = 2\pi R^3/N^2$, and the resulting entropy would be

$$S_{BH} = \frac{2\pi^2 r_+^3}{2\pi R^3/N^2} = \frac{N^2 2\pi^2 R^3 r_+^3}{2\pi R^6} = \pi^2 N^2 T_H^3 \cdot (2\pi R^3). \qquad (51)$$

The radiation formula eq:S-bis applies only to black holes on the ascending branch in Fig. ??, *so that small black holes on the descending branch of the curve would either not radiate, or radiate with some other spectrum than a black body.* But Hawking [?] found precisely that small black holes with temperature given by eq:piccolo do radiate with a thermal spectrum.

However, it was previously asked how the horizon size R is determined by the number of 'branes,' so the two appear to be related. [?, p. 542] It is also ironical to see that while string theorists hold to black body radiations in 3 dimensions, they claim that 5 dimensions are needed. Long ago, Lord Rayleigh [?] gave a generalization to Stefan's law in the case the spatial dimension would be different than 3. By generalizing the equation of state between the pressure p and internal energy density ε, $p = \varepsilon$ to $p = \varepsilon/\eta$, where $\eta > 3$ for dimensions higher than 3, Rayleigh generalized Stefan's law to $\varepsilon \sim T^{1+\eta}$, and, consequently, the entropy density varies as $s \sim T^\eta$. So one cannot retain black body radiation in 3 dimensions, eq:S-bis, while, at the same time, requiring higher spatial dimensions.

5 Regge Scattering Amplitudes and Automorphic Functions

An automorphic function is an extension of the concept of a periodic function to the more general case of discontinuous groups. That is, circular functions are automorphic with respect to the group $\{2\pi n | n \in Z\}$, and the elliptic functions are automorphic with respect to the group $\{m\omega_1 + n\omega_2 | m, n \in Z\}$, with $\omega_1/\omega_2 \notin R\}$. Circular functions are associated with a sphere, a solid body of genus 0. It has zero moduli and consists in transforming a sphere in an infinite number of ways. Elliptic functions are of genus $g = 1$ with one modulus, the ratio ω_1/ω_2. For $g > 1$ there will be $3g - 3$ moduli. Moduli are those constants which play the role of invariants in a uniform transformation from the z- to the ω- complex planes under the mapping $f(\omega, z) = 0$. For elliptic functions, the ω-plane is tessellated by parallelograms.

All this was known before 1880. The modular function, f, defined on the half-plane $\mathrm{Im}(z) > 0$ was known to be automorphic with respect to the modular group of transformations,

$$z \longmapsto \frac{az + b}{cz + d},$$

where $a, b, c, d \in Z$ and $ad - bc = 1$. If the coefficients satisfy these conditions then

$$f\left(\frac{az + b}{cz + d}\right) = f(z).$$

The modular group arose from considerations about the equivalence of birational quadratic forms, and was known already to Gauss in this context. The generators of the modular group are the translations and inversions, $\mathrm{T}_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, and $T_i = \begin{pmatrix} 0 & 1 \end{pmatrix}$

The Riemann space is one of triangular tessellations where the angles of the triangle are related to the exponents of the indicial equations about

[width=0.3]banana.jpeg

Figure 4: Two circular arcs cut each other at an angle $\lambda\pi$ to form a crescent.

Why string theory got started at all was Veneziano's [?] Euler beta integral for the expression of the relativistic four point scattering amplitude according to Regge theory. The Regge trajectories determine the exponents in the beta integral, and the mapping was from the half-plane of poles located on the real axis of the z-plane to the Riemann ω-plane comprised of non-overlapping triangles that tessellate the disc completely. The poles in the z-plane determine the angles of the triangles in the ω-plane.

As an illustration, we will derive the leading term in the scattering amplitude of four particles $a + b \longrightarrow c + d$, known as the Regge limit. Following convention, we introduce the Mandelstam variables s=-($p_a + p_b)^2 = -(p_c + p_d)^2$,
$t = -(p_a - p_c)^2 = -(p_b - p_d)^2$, which apart from sign, are the total energy and momentum transfer, respectively. By defining the crossed momentum transfer, $u = -(p_a - p_d)^2$, it is easy to see that the sum of the Mandelstam variables,

$$s + t + u = 4M^2,$$

gives four times the square of the mass of each particle if the particles have the same mass. We will consider the momentum transfer, t, to be

at $s = s_0$ and $s = \infty$. This corresponds to a single particle threshold.

at $s = \infty$, $z = \infty$. In the example this will mean it will be that arcs with a

only differential equations, of which [illegible], for which the [illegible]

nents of the two solutions in the immediate vicinity of the poles uniquely

[illegible]

since the angle at infinity never exceeds about.

To motivate the physical interpretation of eq:hyper, consider the Schrödinger equation at small radial values, z, for which only the centrifugal barrier dominates,

$$\frac{d^2\psi}{dz^2} - \frac{\ell(\ell+1)}{z^2}\psi = 0, \tag{53}$$

where ℓ is the angular momentum. Now, eq:Schrodinger can be cast in the form eq:hyper by writing

$$\lambda = 2\ell + 1. \tag{54}$$

But, the condition that the angle, $\lambda\pi$ be less than π implies

$$0 \geq \ell \geq -\frac{1}{2}. \tag{55}$$

In this case, $P_\ell(\cos\vartheta)$ will no longer be a Legendre polynomial, but, rather, a hypergeometric function with a branch cut going from -1 to $-\infty$.

The two independent solutions to the equation eq:hyper are

$$w_1 = z^{1+\ell} \qquad \text{and} \qquad w_2 = z^{-\ell}. \tag{56}$$

Consider now the quotient of the solutions

$$\omega = \frac{w_1}{w_2} = \lambda \int_0^z z^{\lambda-1} dz. \tag{57}$$

The second equality shows that ω is the Schwarz-Christoffel mapping function of part of the Veneziano amplitude for the four point elastic scattering amplitude, where $\lambda - 1$ would be an exterior angle of one of the vertices.[2] If there was an additional fixed point at $z = 1$, eq:quotient would have been an incomplete β-function, known as a Schwarz triangle function in the ω-plane. [?]

The linear fractional transformation,

$$\omega' = \frac{a\omega + b}{c\omega + d}, \tag{58}$$

[2]This has been done in [?]. In order to do so it was necessary to identify the angles with $1 - \mathrm{Re}\ \alpha$, which does not cover the zones of bound states and resonances [cf. Fig. ??].

where the coefficients, $a, b, c, \in Z$, $ad - bc = 1$, and $\omega' = \omega'_1/\omega'_2$, will transform two circles that cut one another into two other circles that intersect at the same angles. In other words, the Möbius transform eq:Mob is conformal.

If the ω-circles intersect in $-i$ and $+i$ corresponding to the origin and the point at infinity in the z-plane, then the elliptic substitution,

$$\omega' = \frac{\cos\varphi z + \sin\varphi}{-\sin\varphi z + \cos\varphi}, \tag{59}$$

where $\varphi = \lambda\pi/2$ is a periodic substitution of period $2/\lambda$ which, by repeated application to the area of the crescent, will divide the plane into $2/\lambda$ regions, that, with the exception of two, have the same crescent form [?, p. 630].

On the other hand, if the vertices of the cresent are at -1 and $+1$ in the ω-plane, corresponding to the fixed points 0 and ∞, respectively, the hyperbolic substitution, [3]

$$\omega' = \frac{\cosh\Phi z + \sinh\Phi}{\sinh\Phi z + \cosh\Phi}, \tag{60}$$

where $\Phi = \lambda\pi/2$, will stretch the symmetrically the unit disc D in the ω-plane away from the source at -1 towards the sink at $+1$. Hence, the transform from an elliptic to hyperbolic substitution is tantamount to making the angle imaginary. The trajectories appear as lines of force on the left-hand side of Fig. ??, where the lines start from the positive charge and end on the negative charge. The fundamental region is the crescent with equal angles, $\lambda\pi$. The lines of electrical force are, therefore, actually tessellations of the hyperbolic plane!

To determine the elastic scattering amplitude, we observe that when t is small and $s \to \infty$, $\cos\vartheta \propto -s$, and it can be shown that [?]

$$P_\ell(-\cos\vartheta) \propto s^\ell = e^{\ell(\ln s)} \qquad \text{for} \qquad \text{Re}\,\ell > -. \tag{61}$$

Since eq:Legendre is proportional to the scattering amplitude, In order for there to be an exponential falloff in the elastic scattering amplitude

[3] It is a great pity that Poincaré, who developed the theory of automorphic functions, [?] did not appreciate that the same type of transforms, later to become known as Lorentz transforms, did not apply to relativity.

[width=0.5]trajectory.jpeg

Figure 5: A Regge trajectory with even signature spanning three regions: I s-channel where s has the meaning of total energy, II the bound state region, and III the resonance region.

with increasing momentum transfer, $\ell = \alpha(t) < 0$. This together with the condition in eq:Legendre is precisely the interval eq:ell.

A Regge trajectory takes the linear form

$$\ell = \alpha(t) = \alpha' t + \alpha_0, \tag{62}$$

as shown in Fig. ?? For negative values of eq:trajectory, t has the significance of momentum transfer and s of total energy. Thus, eq:trajectory governs high energy behavior in the s-channel. The trajectory contributes to the power behavior, eq:Legendre, of the s-channel amplitude. In this region $\alpha(t)$ can be detected for all values of $t < 0$.

As t crosses into the positive region, it takes on the meaning of total energy, and $\alpha(t)$ becomes observable only at integers differing by values of 2 (even signature), as shown in Fig. ??. The values of t that make $\alpha(t)$ pass through non-negative integers ℓ correspond to bound states (II), or resonances (III), of angular momentum ℓ,

$$t = M_\ell^2 = \frac{\ell - \alpha_0}{\alpha'}, \qquad \ell = 0, 2, 4, \ldots \qquad (\alpha_0 < 0). \tag{63}$$

For $t < t_T$ (the threshold), $\ell = 0$ corresponds to a bound state of imaginary mass M_0 having zero spin, i.e. a tachyon. Moreover, there are resonances of masses M_2 and M_4 having spins 2 and 4, respectively.

It is therefore apparent that automorphic functions are related to continuous, negative, values of ℓ, while discrete, positive, values of ℓ violate the condition that the angle eq:lambda be less than 1. That is, there is a discrete fundamental region for a continuous range of values of ℓ given by eq:ell.

If the total cross-section is to be nearly constant at high s, it needs $\alpha_0 = 1$, which is the limit for the strong interaction under crossing [?, p. 183]. This trajectory is called a Pomeron, and was used to account for the asymptotic behavior of the total cross-section. This is precisely the trajectory the string theorists use, and it would make the zero angular momentum state a tachyon according to eq:M. Since they do not impose changes

in angular momentum in intervals of 2, the $\ell = 1$ would be massless [?, p. 626]. Particles of new masses, M_ℓ, would appear for ever increasing non-negative integer values of the angular momentum. However, all this has nothing whatsoever to do with a vibrating flexible string.

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THE PRINCIPLE OF LEAST ACTION IN COVARIANT THEORY OF GRAVITATION

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Abstract

The integral equations for calculation of metric, equations of substance motion, as well as equations for gravitational and electromagnetic fields in covariant theory of gravitation are obtained by means of variation of action functional. In covariant form stress-energy tensor of gravitational field, strength tensor of gravitational field and 4-current of mass are determined. The meaning of the cosmological constant and its relation to the components of energy density in action functional are explained. The results obtained prove the validity of Mach's principle, assuming that gravitation effects are due to the flows of gravitons. The idea that metric can be entirely determined by variables describing fields' properties is substantiated.

Keywords: action; metric; cosmological constant; stress-energy tensor of gravitational field; equations of motion; field equations; covariant theory of gravitation.

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1 Introduction

Covariant theory of gravitation (CTG) is one of the alternative gravitation theories in relation to the general relativity theory. The purpose of this article is to derive equations of CTG from the principle of least action. As the basis of our discussion we will use works of Einstein [1], Dirac [2], Pauli [3], Fock [4], Landau and Lifshitz [5].

We will use international system of units, basic coordinates in the form of coordinates with contravariant indices (x^0, x^1, x^2, x^3), metric tensor $g_{\mu\nu}$, metric signature (+, –, –, –). The presence of repeated indices in formulas implies Einstein summation convention, which is a separate summation for each repeated index.

2 The action function

In the case of continuously distributed throughout the volume of space matter, the action function for the matter in gravitational and electromagnetic fields in covariant theory of gravitation can be presented as follows:

$$S = \int L\,dt = \int\int\left(k(R - 2\Lambda) - c\rho_0 - \frac{1}{c} D_\mu J^\mu + \frac{c}{16\pi\gamma} \quad {}^{\mu\nu} \quad {}^{\mu\nu} - \right.$$
$$\left. - \frac{1}{c} A_\mu j^\mu - \frac{c\varepsilon_0}{4} F_{\mu\nu} F^{\mu\nu} \right) \sqrt{-g}\, d\Sigma, \tag{1}$$

where L – Lagrange function or Lagrangian,

dt – differential of time in the used reference frame,

k – certain coefficient,

R – scalar curvature,

Λ – a constant, which characterizes the energy density of the system as a whole, and therefore is a function of the system,

c – speed of light, as a measure of velocity of electromagnetic and gravitational interactions propagation,

ρ_0 – density of substance mass in reference frame in which the substance is at rest,

$D_\mu = \left(\frac{\psi}{c}, -\boldsymbol{D}\right)$ – 4-potential of gravitational field which is described by scalar potential ψ and vector potential $\boldsymbol{D}$ of the field,

J^μ – 4-vector of mass current,

γ – gravitational constant,

$\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$ – gravitational tensor (tensor of gravitational field strength),

$\Phi^{\alpha\beta} = g^{\alpha\mu} g^{\nu\beta} \Phi_{\mu\nu}$ – determining the gravitational tensor with contravariant indices by means of the metric tensor $g^{\alpha\mu}$,

$A_\mu = \left(\frac{\varphi}{c}, -\boldsymbol{A}\right)$ – 4-potential of electromagnetic field, set by scalar potential φ and vector potential $\boldsymbol{A}$ of the field,

j^μ – 4-vector of electric current density,

ε_0 – electric constant (vacuum permittivity),

$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$ – electromagnetic tensor (field strength tensor),

$\sqrt{-g}\, d\Sigma = \sqrt{-g}\, c\, dt\, dx^1\, dx^2\, dx^3$ – invariant 4-volume, expressed through differential of time coordinate $dx^0 = c\,dt$, through product of differentials of spatial coordinates $dx^1\, dx^2\, dx^3$, and through the square root $\sqrt{-g}$ of determinant g of metric tensor, taken with the negative sign.

The symbol ∇_μ denotes covariant derivative with respect to coordinates (in this case the coordinates x^μ). Similarly, $\partial_\mu = \frac{\partial}{\partial x^\mu}$ is an operator of partial derivative with respect to coordinates or 4-gradient.

The integrand in (1) is Lagrangian function, consisting of six terms. The first term with the scalar curvature R depends on the metric tensor and its derivatives with respect to coordinates. In covariant theory of gravitation (CTG) metric is used to consider the impact of fundamental fields (which include electromagnetic and gravitational fields) of material bodies on the results of space-time measurements near the bodies. The field's effect on

measurement results shows that under the field's action electromagnetic waves are deflected from rectilinear motion, electromagnetic clock changed its course, and measured distances changed its value. These effects can be described by introducing a curved space-time with metric tensor $g_{\mu\nu}$ instead of flat Minkowski space with its single metric tensor $\eta_{\mu\nu}$. In CTG gravitational field is an independent physical field and the metric tensor $g_{\mu\nu}$ has the geometric meaning and auxiliary function, unlike general relativity where the metric field completely replaces gravitational field.

In CTG second term in (1) is not simply related to rest energy substance density and its inertia with respect to the applied forces. According to [6], [7], the rest mass (and substance density at rest) is a consequence of strong gravitation and electromagnetic interactions operating at the level of elementary particles. But the first and the second terms in (1) are associated with microscopic fundamental fields, while other terms refer to action of macroscopic gravitational and electromagnetic fields. The division to microscopic and macroscopic fundamental fields follows from the theory of infinite nesting of matter in which its own gravitational field operates at every main level of matter. As a result, the usual gravitation is assumed as long-range component of strong gravitation.

The third term in (1) $-\frac{1}{c}D_\mu J^\mu$ is invariant with respect to different types of coordinate transformations that reflect the interaction of mass current density $J^\mu = \rho_0 u^\mu$ of arbitrary substance unit with gravitational field.

According to [8], the fourth term in (1) associated with energy field, is an invariant of gravitational field which does not change its form by changing the reference system. The fifth and sixth terms, for electromagnetic field, are similar in structure to the third and fourth terms for gravitational field. And 4-vector of electric current density j^μ can be determined by charge density ρ_{0q} of substance unit and 4-velocity: $j^\mu = \rho_{0q} u^\mu$.

In CTG 4-potentials D_μ and A_μ with covariant indices, and 4-currents J^μ and j^μ with contravariant indices were determined in [7] and [9] as initial concepts for construction of axiomatic theory. Hence we can make a

conclusion that, for example, 4-vector $D^{\mu} = g^{\mu\nu} D_{\nu}$ can not be found in absence of information about metric in any frame of reference.

3 Variation of curvature invariant

To obtain equations for metric the variation of action function for the case when in Lagrangian the metric tensor $g_{\mu\nu}$ is variable should be set to zero. At the same time variation of metric tensor should be zero on the borders of four-dimensional volume for which in (1) integration is performed. For full variation of the action should be as follows:

$$\delta S = \delta \int \Big(k(R - 2\Lambda) - c\rho_0 - \frac{1}{c} D_{\mu} J^{\mu} + \frac{c}{16\pi\gamma} \quad {}^{\mu\nu} - \\ - \frac{1}{c} A_{\mu} j^{\mu} - \frac{c\varepsilon_0}{4} F_{\mu\nu} F^{\mu\nu} \Big) \sqrt{-g}\, d\Sigma = 0. \tag{2}$$

Let's find the variation associated with the first term in (2). Using definition of scalar curvature R through Christoffel symbols, we obtain the same as in [2]:

$$\delta S_1 = \delta \int k(R - 2\Lambda) \sqrt{-g}\, d\Sigma = \\ = k\delta \int R_1 \sqrt{-g}\, d\Sigma + k\delta \int R_2 \sqrt{-g}\, d\Sigma - 2\Lambda k \int \delta \sqrt{-g}\, d\Sigma, \tag{3}$$

where $R_1 = g^{\mu\nu} \left(\partial_{\kappa} \Gamma^{\kappa}_{\mu\nu} - \partial_{\nu} \Gamma^{\kappa}_{\mu\kappa} \right)$, $R_2 = g^{\mu\nu} \left(\Gamma^{\kappa}_{\mu\nu} \Gamma^{\sigma}_{\kappa\sigma} - \Gamma^{\kappa}_{\mu\sigma} \Gamma^{\sigma}_{\kappa\nu} \right)$, $R = R_1 + R_2$.

The expression for $R_1 \sqrt{-g}$ can be obtained by means of differentiation by parts:

$$R_1 \sqrt{-g} = \partial_{\kappa} \left(g^{\mu\nu} \Gamma^{\kappa}_{\mu\nu} \sqrt{-g} \right) - \partial_{\nu} \left(g^{\mu\nu} \Gamma^{\kappa}_{\mu\kappa} \sqrt{-g} \right) - \\ - \Gamma^{\kappa}_{\mu\nu} \partial_{\kappa} \left(g^{\mu\nu} \sqrt{-g} \right) + \Gamma^{\kappa}_{\mu\kappa} \partial_{\nu} \left(g^{\mu\nu} \sqrt{-g} \right) \tag{4}$$

The first two terms on the right side of (4) are the total derivatives (divergence), and after substituting them in (3), the integrals of the divergence over the volume according to Gauss's theorem can be replaced by the integrals over the surface surrounding the volume for which the integration takes place. Since variation of metric tensor on the surface equals to zero, these terms will not contribute to the variation of action function, so that in (4) only the last two terms should be taken into account. Then we can use two relations:

$$\partial_\kappa\left(g^{\mu\nu}\sqrt{-g}\right)=\sqrt{-g}\left(g^{\mu\nu}\Gamma^{\alpha}_{\kappa\alpha}-g^{\nu\beta}\Gamma^{\mu}_{\beta\kappa}-g^{\mu\alpha}\Gamma^{\nu}_{\alpha\kappa}\right), \tag{5}$$

$$\partial_\nu\left(g^{\mu\nu}\sqrt{-g}\right)=-\sqrt{-g}\,g^{\nu\beta}\Gamma^{\mu}_{\beta\nu}.$$

Substituting them in the last two terms in (4) and renaming some of indices on which the summation takes place, we obtain:

$$\begin{aligned}
&-\Gamma^{\kappa}_{\mu\nu}\sqrt{-g}\left(g^{\mu\nu}\Gamma^{\alpha}_{\kappa\alpha}-g^{\nu\beta}\Gamma^{\mu}_{\beta\kappa}-g^{\mu\alpha}\Gamma^{\nu}_{\alpha\kappa}\right)-\Gamma^{\kappa}_{\mu\kappa}\sqrt{-g}\,g^{\nu\beta}\Gamma^{\mu}_{\beta\nu}=\\
&=-\sqrt{-g}\left(g^{\mu\nu}\Gamma^{\alpha}_{\kappa\alpha}\Gamma^{\kappa}_{\mu\nu}-g^{\nu\beta}\Gamma^{\mu}_{\beta\kappa}\Gamma^{\kappa}_{\mu\nu}-g^{\mu\alpha}\Gamma^{\nu}_{\alpha\kappa}\Gamma^{\kappa}_{\mu\nu}+g^{\nu\beta}\Gamma^{\mu}_{\beta\nu}\Gamma^{\kappa}_{\mu\kappa}\right)=\\
&=-2\sqrt{-g}\left(g^{\mu\nu}\Gamma^{\alpha}_{\kappa\alpha}\Gamma^{\kappa}_{\mu\nu}-g^{\nu\beta}\Gamma^{\mu}_{\beta\kappa}\Gamma^{\kappa}_{\mu\nu}\right)=-2\sqrt{-g}\,g^{\mu\nu}\left(\Gamma^{\kappa}_{\mu\nu}\Gamma^{\sigma}_{\kappa\sigma}-\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\right)=\\
&=-2R_2\sqrt{-g}.
\end{aligned}$$

As a result, instead of (3) we can write:

$$\delta S_1=-k\int\delta\left(R_2\sqrt{-g}\right)d\Sigma-2\Lambda k\int\delta\sqrt{-g}\,d\Sigma. \tag{6}$$

The first variation in (6) will equal to:

$$\delta\left(R_2\sqrt{-g}\right)=\delta\left(\Gamma^{\kappa}_{\mu\nu}\Gamma^{\sigma}_{\kappa\sigma}g^{\mu\nu}\sqrt{-g}\right)-\delta\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}g^{\mu\nu}\sqrt{-g}\right). \tag{7}$$

Using the relation: $\Gamma^{\sigma}_{\kappa\sigma} = \frac{1}{\sqrt{-g}}\partial_{\kappa}\sqrt{-g}$, differentiation by parts, and using the second relation in (5), for the first part (7) we obtain:

$$\delta\left(\Gamma^{\kappa}_{\mu\nu}\Gamma^{\sigma}_{\kappa\sigma}g^{\mu\nu}\sqrt{-g}\right) = \Gamma^{\kappa}_{\mu\nu}\delta\left(\Gamma^{\sigma}_{\kappa\sigma}g^{\mu\nu}\sqrt{-g}\right) + \Gamma^{\sigma}_{\kappa\sigma}g^{\mu\nu}\sqrt{-g}\,\delta\Gamma^{\kappa}_{\mu\nu} =$$
$$= \Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\partial_{\kappa}\sqrt{-g}\right) + \Gamma^{\sigma}_{\kappa\sigma}\delta\left(g^{\mu\nu}\Gamma^{\kappa}_{\mu\nu}\sqrt{-g}\right) - \Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right) =$$
$$= \Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\partial_{\kappa}\sqrt{-g}\right) - \Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right) - \Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right).$$

(8)

The expression for derivative of metric tensor has the form: $\partial_{\alpha}g^{\nu\beta} = -g^{\beta\kappa}\Gamma^{\nu}_{\kappa\alpha} - g^{\nu\kappa}\Gamma^{\beta}_{\kappa\alpha}$. After multiplication by $\sqrt{-g}$, taking variation and another multiplication by $\Gamma^{\alpha}_{\nu\beta}$ we will obtain:

$$\delta\left(\sqrt{-g}\,\partial_{\alpha}g^{\nu\beta}\right)\Gamma^{\alpha}_{\nu\beta} = -\delta\left(g^{\beta\kappa}\Gamma^{\nu}_{\kappa\alpha}\sqrt{-g}\right)\Gamma^{\alpha}_{\nu\beta} - \delta\left(g^{\nu\kappa}\Gamma^{\beta}_{\kappa\alpha}\sqrt{-g}\right)\Gamma^{\alpha}_{\nu\beta} =$$
$$= -2\delta\left(\Gamma^{\nu}_{\kappa\alpha}g^{\beta\kappa}\sqrt{-g}\right)\Gamma^{\alpha}_{\nu\beta}.$$

We now transform the second part of (7), using replacing of indices, operation of differentiation by parts, and the previous expression:

$$\delta\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}g^{\mu\nu}\sqrt{-g}\right) = \delta\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\right)g^{\mu\nu}\sqrt{-g} + \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right) =$$
$$= 2\delta\left(\Gamma^{\kappa}_{\mu\sigma}\right)\Gamma^{\sigma}_{\kappa\nu}g^{\mu\nu}\sqrt{-g} + \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right) =$$
$$= 2\delta\left(\Gamma^{\kappa}_{\mu\sigma}g^{\mu\nu}\sqrt{-g}\right)\Gamma^{\sigma}_{\kappa\nu} - \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right) =$$
$$= -\delta\left(\sqrt{-g}\,\partial_{\sigma}g^{\kappa\nu}\right)\Gamma^{\sigma}_{\kappa\nu} - \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right).$$

(9)

Substitution of (8) and (9) in (7) gives the following result:

$$\delta\left(R_2\sqrt{-g}\right)=\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\partial_{\kappa}\sqrt{-g}\right)-\Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right)-$$
$$-\Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)+\delta\left(\sqrt{-g}\,\partial_{\sigma}g^{\kappa\nu}\right)\Gamma^{\sigma}_{\kappa\nu}+\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)=$$
$$=\Gamma^{\kappa}_{\mu\nu}\delta\left(\partial_{\kappa}\left(g^{\mu\nu}\sqrt{-g}\right)\right)-\Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right)+$$
$$+\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}-\Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\right)\delta\left(g^{\mu\nu}\sqrt{-g}\right). \tag{10}$$

The terms $\Gamma^{\kappa}_{\mu\nu}\delta\left(\partial_{\kappa}\left(g^{\mu\nu}\sqrt{-g}\right)\right)$ and $\Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right)$ in (10) can be transformed:

$$\Gamma^{\kappa}_{\mu\nu}\delta\left(\partial_{\kappa}\left(g^{\mu\nu}\sqrt{-g}\right)\right)=\Gamma^{\kappa}_{\mu\nu}\partial_{\kappa}\delta\left(g^{\mu\nu}\sqrt{-g}\right)=$$
$$=\partial_{\kappa}\left(\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)\right)-\left(\partial_{\kappa}\Gamma^{\kappa}_{\mu\nu}\right)\delta\left(g^{\mu\nu}\sqrt{-g}\right). \tag{11}$$

$$\Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_{\nu}\left(g^{\kappa\nu}\sqrt{-g}\right)\right)=\Gamma^{\sigma}_{\kappa\sigma}\partial_{\nu}\delta\left(g^{\kappa\nu}\sqrt{-g}\right)=$$
$$=\partial_{\nu}\left(\Gamma^{\sigma}_{\kappa\sigma}\delta\left(g^{\kappa\nu}\sqrt{-g}\right)\right)-\left(\partial_{\nu}\Gamma^{\sigma}_{\kappa\sigma}\right)\delta\left(g^{\kappa\nu}\sqrt{-g}\right).$$

In equations (11) there are such divergences as $\partial_{\kappa}\left(\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)\right)$, which, after substituting in (10) and then into (6) will be integrated over 4-volume and transformed into integrals over the surface, where variations are equal to zero. With this in mind, after substituting (10) and (11) into (6) we obtain:

$$
\begin{aligned}
&-k\int\delta\left(R_2\sqrt{-g}\right)d\Sigma= \\
&=-k\int\left(\partial_\nu\Gamma^\sigma_{\mu\sigma}-\partial_\kappa\Gamma^\kappa_{\mu\nu}+\Gamma^\kappa_{\mu\sigma}\Gamma^\sigma_{\kappa\nu}-\Gamma^\sigma_{\kappa\sigma}\Gamma^\kappa_{\mu\nu}\right)\delta\left(g^{\mu\nu}\sqrt{-g}\right)d\Sigma= \\
&=k\int R_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)d\Sigma,
\end{aligned}
\tag{12}
$$

where $R_{\mu\nu}$ is Ricci tensor.

For variations of metric tensor $g^{\mu\nu}$ and $\sqrt{-g}$ we can write down:

$$
\delta g^{\mu\nu}=-g^{\mu\alpha}g^{\nu\beta}\delta g_{\alpha\beta}, \qquad \delta\sqrt{-g}=\frac{\sqrt{-g}}{2}g^{\alpha\beta}\delta g_{\alpha\beta}. \tag{13}
$$

Using (13) in (12), we obtain:

$$
\begin{aligned}
&-k\int\delta\left(R_2\sqrt{-g}\right)d\Sigma=k\int R_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)d\Sigma= \\
&=k\int R_{\mu\nu}\left(\sqrt{-g}\,\delta g^{\mu\nu}+g^{\mu\nu}\delta\sqrt{-g}\right)d\Sigma= \\
&=k\int R_{\mu\nu}\left(-g^{\mu\alpha}g^{\nu\beta}+g^{\mu\nu}\frac{1}{2}g^{\alpha\beta}\right)\sqrt{-g}\,\delta g_{\alpha\beta}d\Sigma= \\
&=k\int\left(-R^{\alpha\beta}+\frac{1}{2}g^{\alpha\beta}R\right)\sqrt{-g}\,\delta g_{\alpha\beta}d\Sigma.
\end{aligned}
$$

With this result and the expression $\delta\sqrt{-g}$ from (13), we obtain:

$$
\delta S_1=k\int\left(-R^{\alpha\beta}+\frac{1}{2}g^{\alpha\beta}R-\Lambda g^{\alpha\beta}\right)\sqrt{-g}\,\delta g_{\alpha\beta}d\Sigma. \tag{14}
$$

4 Variation of invariant mass density

The second term in (2) is an invariant associated with the 4-current of mass J^μ, so we can write down:

$$c\rho_0 = \sqrt{g_{\mu\nu} J^\mu J^\nu} = \sqrt{J_\nu J^\nu}\,. \tag{15}$$

4-vector J^μ can be determined through 4-velosity $u^\mu = \frac{dx^\mu}{d\tau}$, where $dx^\mu = (dx^0, dx^1, dx^2, dx^3)$ is displacement 4-vector, $d\tau$ – differential of proper time, as follows: $J^\mu = \rho_0 u^\mu = \frac{u^\mu \sqrt{g_{\alpha\beta} J^\alpha J^\beta}}{c}$, and $u_\beta u^\beta = c^2$. In elementary particle physics instead of the values of mass and velocity of particles their energies E and momentums $\boldsymbol{p}$ are used as the quantities directly found from experiments. These quantities are part of 4-momentum of a particle: $p^\alpha = \left(\frac{E}{c}, \boldsymbol{p}\right) = mu^\alpha$, and the invariant mass m become a secondary concept, which may be found from the relation $\sqrt{p_\beta p^\beta} = mc = \frac{\sqrt{E^2 - p^2 c^2}}{c}$. Accordingly, to calculate the velocity of particles in special relativity theory is applied relation: $\boldsymbol{v} = \frac{c^2}{E}\boldsymbol{p}$.

The variation of the second term in (2) with (15) has the following form:

$$\delta S_2 = -\int \delta\left(\sqrt{g_{\mu\nu} J^\mu J^\nu}\sqrt{-g}\right) d\Sigma. \tag{16}$$

We determine the variation in (16) with the help of (13):

$$\begin{aligned}
&\delta\left(\sqrt{g_{\mu\nu} J^\mu J^\nu}\sqrt{-g}\right) = \sqrt{-g}\,\delta\left(\sqrt{g_{\mu\nu} J^\mu J^\nu}\right) + \sqrt{g_{\mu\nu} J^\mu J^\nu}\,\delta\sqrt{-g} = \\
&= \frac{J^\mu J^\nu \sqrt{-g}\,\delta g_{\mu\nu}}{2\sqrt{g_{\mu\nu} J^\mu J^\nu}} + \frac{g_{\mu\nu} J^\mu \sqrt{-g}\,\delta J^\nu}{\sqrt{g_{\mu\nu} J^\mu J^\nu}} + \frac{1}{2}\sqrt{g_{\mu\nu} J^\mu J^\nu}\sqrt{-g}\, g^{\alpha\beta}\delta g_{\alpha\beta} = \\
&= \frac{1}{2}\left(\frac{J^\alpha J^\beta}{\sqrt{g_{\mu\nu} J^\mu J^\nu}} + g^{\alpha\beta}\sqrt{g_{\mu\nu} J^\mu J^\nu}\right)\sqrt{-g}\,\delta g_{\alpha\beta} + \frac{g_{\alpha\beta} J^\alpha}{\sqrt{g_{\mu\nu} J^\mu J^\nu}}\sqrt{-g}\,\delta J^\beta.
\end{aligned} \tag{17}$$

In (17) variation δJ^{β} is contained which according to [2], [4] can be found using displacement 4-vector ξ^{μ}. The displacement ξ^{μ} is variation of coordinates, which gives variation of mass 4-current J^{β}:

$$\delta J^{\beta} = \nabla_{\sigma}\left(J^{\sigma}\xi^{\beta} - J^{\beta}\xi^{\sigma}\right) = \frac{1}{\sqrt{-g}}\partial_{\sigma}\left[\sqrt{-g}\left(J^{\sigma}\xi^{\beta} - J^{\beta}\xi^{\sigma}\right)\right]. \qquad (18)$$

Equation (18) was obtained based on the condition that mass of substance unit in variation of coordinates remains constant despite the change in density and its volume. With the help of (15) and (18) the last term in (17) can be transformed through 4-velosity u_{β}:

$$\frac{g_{\alpha\beta}J^{\alpha}}{\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}}\sqrt{-g}\,\delta J^{\beta} = \frac{1}{c}u_{\beta}\partial_{\sigma}\left[\sqrt{-g}\left(J^{\sigma}\xi^{\beta} - J^{\beta}\xi^{\sigma}\right)\right] =$$

$$= \frac{1}{c}\partial_{\sigma}\left[u_{\beta}\sqrt{-g}\left(J^{\sigma}\xi^{\beta} - J^{\beta}\xi^{\sigma}\right)\right] - \frac{1}{c}\partial_{\sigma}u_{\beta}\sqrt{-g}\left(J^{\sigma}\xi^{\beta} - J^{\beta}\xi^{\sigma}\right).$$

The term with complete divergence in integration over 4-volume in the action function will not make any contribution. The remaining term in the previous equation can be transformed further:

$$-\frac{1}{c}\partial_{\sigma}u_{\beta}\sqrt{-g}\left(J^{\sigma}\xi^{\beta} - J^{\beta}\xi^{\sigma}\right) = -\frac{1}{c}(\partial_{\sigma}u_{\beta} - \partial_{\beta}u_{\sigma})J^{\sigma}\xi^{\beta}\sqrt{-g} =$$

$$= -\frac{1}{c}(\nabla_{\sigma}u_{\beta} - \nabla_{\beta}u_{\sigma})J^{\sigma}\xi^{\beta}\sqrt{-g} = -\frac{1}{c}J^{\sigma}\nabla_{\sigma}u_{\beta}\xi^{\beta}\sqrt{-g}. \qquad (19)$$

Here the condition $J^{\sigma}\nabla_{\beta}u_{\sigma} = \rho_0 u^{\sigma}\nabla_{\beta}u_{\sigma} = 0$ was used, since it follows from the equation $u^{\sigma}u_{\sigma} = c^2$, to which the covariant derivative ∇_{β} is applied.

The symmetrical stress-energy tensor of substance is:

$$\phi^{\alpha\beta} = \frac{c J^\alpha J^\beta}{\sqrt{g_{\mu\nu} J^\mu J^\nu}} . \qquad (20)$$

Substituting (20) in (17) and using (19) instead of the last term in (17), we obtain the variation δS_2 in (16):

$$\delta S_2 = \int\left(-\frac{1}{2c}\phi^{\alpha\beta}\,\delta g_{\alpha\beta} - \frac{1}{2} g^{\alpha\beta}\sqrt{g_{\mu\nu} J^\mu J^\nu}\,\delta g_{\alpha\beta} + \frac{1}{c} J^\sigma \nabla_\sigma u_\beta \xi^\beta \right)\sqrt{-g}\, d\Sigma . \qquad (21)$$

5 Variation of Lagrangian of gravitational field and its sources

The effect of macroscopic gravitational field is revealed in the third and fourth terms in (2), which gives the following:

$$\delta S_3 = -\int \frac{1}{c}\delta\left(D_\mu J^\mu \sqrt{-g}\right) d\Sigma , \quad \delta S_4 = \int \frac{c}{16\pi\gamma}\delta\left(\Phi_{\mu\nu}\Phi^{\mu\nu}\sqrt{-g}\right) d\Sigma . \qquad (22)$$

We first consider variation for δS_3 in (22), using $\delta\sqrt{-g}$ by (13) and then (18) for δJ^μ :

$$\delta\left(D_\mu J^\mu \sqrt{-g}\right) = D_\mu \delta\left(J^\mu \sqrt{-g}\right) + J^\mu \sqrt{-g}\,\delta D_\mu =$$
$$= D_\mu \partial_\sigma\left[\sqrt{-g}\left(J^\sigma \xi^\mu - J^\mu \xi^\sigma\right)\right] + \frac{1}{2} D_\mu J^\mu\, g^{\alpha\beta}\sqrt{-g}\,\delta g_{\alpha\beta} + J^\mu \sqrt{-g}\,\delta D_\mu .$$

Transformation of the first term:

$$D_\mu \partial_\sigma\left[\sqrt{-g}\left(J^\sigma \xi^\mu - J^\mu \xi^\sigma\right)\right] =$$
$$= \partial_\sigma\left[D_\mu \sqrt{-g}\left(J^\sigma \xi^\mu - J^\mu \xi^\sigma\right)\right] - \partial_\sigma D_\mu \sqrt{-g}\left(J^\sigma \xi^\mu - J^\mu \xi^\sigma\right).$$

Neglecting the term with total derivative, we consider the following:

$$-\partial_\sigma D_\mu \sqrt{-g}\left(J^\sigma \xi^\mu - J^\mu \xi^\sigma\right) = -(\partial_\sigma D_\mu - \partial_\mu D_\sigma) J^\sigma \xi^\mu \sqrt{-g} = \Phi_{\mu\sigma} J^\sigma \xi^\mu \sqrt{-g}.$$

Substituting these results into (22), we obtain:

$$\delta S_3 = \int \left(-\frac{1}{c}\Phi_{\beta\sigma} J^\sigma \xi^\beta - \frac{1}{2c} D_\mu J^\mu g^{\alpha\beta} \delta g_{\alpha\beta} - \frac{1}{c} J^\beta \delta D_\beta \right) \sqrt{-g}\, d\Sigma. \qquad (23)$$

Variation for δS_4 in (22) with (13) is equal to:

$$\begin{aligned} &\delta\left(\Phi_{\mu\nu}\Phi^{\mu\nu}\sqrt{-g}\right) = \delta\left(\Phi_{\mu\nu}\Phi^{\mu\nu}\right)\sqrt{-g} + \Phi_{\mu\nu}\Phi^{\mu\nu}\delta\sqrt{-g} = \\ &= \Phi_{\mu\nu}\delta\Phi^{\mu\nu}\sqrt{-g} + \Phi^{\mu\nu}\delta\Phi_{\mu\nu}\sqrt{-g} + \frac{1}{2}\Phi_{\mu\nu}\Phi^{\mu\nu} g^{\alpha\beta}\sqrt{-g}\ \delta g_{\alpha\beta}. \end{aligned} \qquad (24)$$

Since $\Phi^{\mu\nu} = g^{\mu\alpha} g^{\beta\nu} \Phi_{\alpha\beta}$, the tensor $\Phi_{\alpha\beta}$ is antisymmetrical, then using $\delta g^{\beta\nu}$ by (13), we obtain:

$$\begin{aligned} &\Phi_{\mu\nu}\delta\Phi^{\mu\nu}\sqrt{-g} = \Phi_{\mu\nu}\delta\left(g^{\mu\alpha} g^{\beta\nu}\Phi_{\alpha\beta}\right)\sqrt{-g} = \\ &= \Phi_{\mu\nu}\left[g^{\mu\alpha} g^{\beta\nu}\delta\Phi_{\alpha\beta} + g^{\mu\alpha}\Phi_{\alpha\beta}\delta g^{\beta\nu} + g^{\beta\nu}\Phi_{\alpha\beta}\delta g^{\mu\alpha}\right]\sqrt{-g} = \\ &= \Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g} + 2\Phi_{\mu\nu} g^{\mu\alpha}\Phi_{\alpha\beta}\sqrt{-g}\,\delta g^{\beta\nu} = \\ &= \Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g} - 2 g^{\nu\beta}\Phi_{\kappa\nu}\Phi^{\kappa\alpha}\sqrt{-g}\,\delta g_{\alpha\beta}. \end{aligned}$$

Substitution of this expression into (24) gives the following result:

$$\begin{aligned} &\delta\left(\Phi_{\mu\nu}\Phi^{\mu\nu}\sqrt{-g}\right) = \Phi_{\mu\nu}\delta\Phi^{\mu\nu}\sqrt{-g} + \Phi^{\mu\nu}\delta\Phi_{\mu\nu}\sqrt{-g} + \\ &+\frac{1}{2}\Phi_{\mu\nu}\Phi^{\mu\nu} g^{\alpha\beta}\sqrt{-g}\ \delta g_{\alpha\beta} = 2\Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g} - 2 g^{\nu\beta}\Phi_{\kappa\nu}\Phi^{\kappa\alpha}\sqrt{-g}\,\delta g_{\alpha\beta} + \\ &+\frac{1}{2}\Phi_{\mu\nu}\Phi^{\mu\nu} g^{\alpha\beta}\sqrt{-g}\ \delta g_{\alpha\beta}. \end{aligned} \qquad (25)$$

We denote $U^{\alpha\beta}$ as the stress-energy tensor of gravitational field:

$$U^{\alpha\beta} = \frac{c^2}{4\pi\gamma}\left(g^{\alpha\nu}\Phi_{\kappa\nu}\Phi^{\kappa\beta} - \frac{1}{4}g^{\alpha\beta}\Phi_{\mu\nu}\Phi^{\mu\nu} \right) = -\frac{c^2}{4\pi\gamma}\left(\Phi^{\alpha}{}_{\kappa}\Phi^{\kappa\beta} + \frac{1}{4}g^{\alpha\beta}\Phi_{\mu\nu}\Phi^{\mu\nu} \right). \tag{26}$$

Remembering that $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$, using differentiation by parts, as well as equality for an antisymmetrical tensor: $\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\right) = \sqrt{-g}\,\nabla_\alpha\Phi^{\alpha\beta}$, for the term $2\Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g}$ in (25) we obtain:

$$\begin{aligned} &2\Phi^{\alpha\beta}\delta\Phi_{\alpha\beta}\sqrt{-g} = 2\Phi^{\alpha\beta}\delta\left(\partial_\alpha D_\beta - \partial_\beta D_\alpha\right)\sqrt{-g} = 2\Phi^{\alpha\beta}\left(\partial_\alpha\delta D_\beta - \partial_\beta\delta D_\alpha\right)\sqrt{-g} = \\ &= 4\Phi^{\alpha\beta}\sqrt{-g}\,\partial_\alpha\delta D_\beta = 4\,\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta\right) - 4\,\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\right)\delta D_\beta = \\ &= 4\,\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta\right) - 4\,\nabla_\alpha\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta . \end{aligned} \tag{27}$$

The term $4\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta\right)$ in the last expression is divergence and can be neglected for variation of action function. Substituting (26) and (27) in (25), and the result in (22), we obtain:

$$\delta S_4 = \int\left(-\frac{c}{4\pi\gamma}\nabla_\alpha\Phi^{\alpha\beta}\,\delta D_\beta - \frac{1}{2c}U^{\alpha\beta}\delta g_{\alpha\beta} \right)\sqrt{-g}\,d\Sigma\ . \tag{28}$$

6 Variation of Lagrangian function of electromagnetic field and its sources

Variation in (2) for electromagnetic field is the same as for gravitational field in the previous section. For the fifth and sixth terms in (2) we can write down:

$$\delta S_5 = -\int \frac{1}{c}\delta\left(A_\mu\, j^\mu \sqrt{-g}\right) d\Sigma, \qquad \delta S_6 = -\int \frac{c\varepsilon_0}{4}\delta\left(F_{\mu\nu}F^{\mu\nu}\sqrt{-g}\right) d\Sigma. \qquad (29)$$

Replacing in (22) D_μ with A_μ, J^μ with j^μ, $\Phi_{\beta\sigma}$ with $F_{\beta\sigma}$, instead of (23) we obtain:

$$\delta S_5 = \int\left(-\frac{1}{c}F_{\beta\sigma}j^\sigma\xi^\beta - \frac{1}{2c}A_\mu j^\mu g^{\alpha\beta}\delta g_{\alpha\beta} - \frac{1}{c}j^\beta\delta A_\beta\right)\sqrt{-g}\, d\Sigma. \qquad (30)$$

In deriving (30) was used the expression for variation of electromagnetic 4-current, similar to (18):

$$\delta j^\beta = \nabla_\sigma\left(j^\sigma\xi^\beta - j^\beta\xi^\sigma\right) = \frac{1}{\sqrt{-g}}\partial_\sigma\left[\sqrt{-g}\left(j^\sigma\xi^\beta - j^\beta\xi^\sigma\right)\right]. \qquad (31)$$

The stress-energy tensor of electromagnetic field is:

$$W^{\alpha\beta} = \varepsilon_0 c^2\left(-g^{\alpha\nu}F_{\kappa\nu}F^{\kappa\beta} + \frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\right) = \varepsilon_0 c^2\left(F^\alpha{}_\kappa F^{\kappa\beta} + \frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\right). \qquad (32)$$

With the help of this tensor the variation δS_6 will equal as in (28) to:

$$\delta S_6 = \int\left(c\varepsilon_0\nabla_\alpha F^{\alpha\beta}\,\delta A_\beta - \frac{1}{2c}W^{\alpha\beta}\delta g_{\alpha\beta}\right)\sqrt{-g}\, d\Sigma\ . \qquad (33)$$

7 The equations for metric

Let us put together and substitute in (2) all terms in (14), (21), (23), (28), (30) and (33) containing the variation $\delta g_{\alpha\beta}$ of metric tensor. Due to the arbitrariness of the variation the sum of all these terms should equal to zero. The result is the following:

$$k\left(-R^{\alpha\beta}+\frac{1}{2}g^{\alpha\beta}R-\Lambda g^{\alpha\beta}\right)-\frac{1}{2c}\phi^{\alpha\beta}-\frac{1}{2}g^{\alpha\beta}\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}-\frac{1}{2c}D_{\mu}J^{\mu}\,g^{\alpha\beta}-$$

$$-\frac{1}{2c}U^{\alpha\beta}-\frac{1}{2c}A_{\mu}j^{\mu}\,g^{\alpha\beta}-\frac{1}{2c}W^{\alpha\beta}=0.$$

Let us rewrite this equation with $k=-\frac{c^3}{16\pi\gamma\beta}$, where β – the coefficient of order 1, as an equation for determining metric tensor $g^{\alpha\beta}$ with the help of known sources of energy-momentum. Here, instead of Λ, we introduce a new constant χ, according to the relation: $\Lambda=\frac{8\pi\gamma\beta\chi}{c^4}$. As well as Λ, the constant χ determines the properties of system as a whole. It has the following result:

$$R^{\alpha\beta}-\frac{1}{2}g^{\alpha\beta}R=\frac{8\pi\gamma\beta}{c^4}\left(\phi^{\alpha\beta}+U^{\alpha\beta}+W^{\alpha\beta}+c\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}\,g^{\alpha\beta}+\right.$$
$$\left.+D_{\mu}J^{\mu}\,g^{\alpha\beta}+A_{\mu}j^{\mu}\,g^{\alpha\beta}-\chi g^{\alpha\beta}\right). \qquad (34)$$

In case when such a large system, like our universe is considered Λ has a special name – the cosmological constant. It is estimated as $10^{-52}\,\mathrm{m}^{-2}$. Hence the value χ is of the order $\frac{c^4\Lambda}{8\pi\gamma\beta}\approx 5\cdot 10^{-10}$ J/m^3, having dimension of energy density. For other systems, which can roughly be considered as systems with the matter continuously distributed throughout the volume of space, the constants Λ and χ can have other values.

The equation (34) was obtained based on the condition that either the variations of coordinates ξ^{β} and 4-potentials δD_{β} and δA_{β} in function of action (2) equal to zero, or the sums of all the terms with these variations are always equal to zero. In the first case (34) is an equation for the metric of a system, in which the motion of charged and gravitating substance by the definite trajectories were originally determined and the calibrated values of field potentials (that means, the constants appearing in potentials were determined) were determined as well. In the second case variations of

coordinates (trajectories of substance) and variations of potentials are allowed due to their mutual influence on each other. However, it is assumed that each time the combinations of terms in function of action defining the relationship between the substance and the field, including generation of fields by substance and field influence on substance are of such kind that they equal to zero and do not affect the function of action and the metric. In the second case the initial distribution of substance in space and its initial velocity and initial values of the potentials are arbitrarily, so the laws of connection between the subsequent motion of substance and fields due to some reasons lead to the equation (34). Obviously, the validity of the second case requires additional evidence, or should be postulated, whereas in the former case it is not required.

Outside the substance, where gravitational and electromagnetic 4-currents J^{μ} and j^{μ} tend to zero, the contribution to the metric according to (34) is made only by the stress-energy tensor of gravitational field $U^{\alpha\beta}$ (26) and the stress-energy tensor of electromagnetic field $W^{\alpha\beta}$ (32). If the metric is determined within the substance, the contribution to the metric depends on all terms in (34).

We should note that the right side of (34) contains additional terms that in general relativity theory usually are not considered. In particular, (34) includes all invariant scalar values from the function (1), including terms $\Phi_{\mu\nu}\Phi^{\mu\nu}$ and $F_{\mu\nu}F^{\mu\nu}$, which became components of the tensors $U^{\alpha\beta}$ and $W^{\alpha\beta}$, respectively. In general relativity, there is no stress-energy tensor of gravitational field $U^{\alpha\beta}$ in the form in which we have determined it. This follows from the fact that in general relativity, gravitational field reduces to the metric field, where the components $g_{\mu\nu}$ of metric tensor are considered as potentials describing the gravitational field. In this case, the presence of $U^{\alpha\beta}$ in the right side of equation for the metric (34) would mean that gravitational field is the source itself. In the absence of substance that would lead to a vicious circle where a metric gravitational field generates itself, the field gives the metric and the metric gives the field. As opposed to it, in covariant theory of gravitation (CTG), the metric is only an auxiliary geometric field induced by gravitation and electromagnetic field, taken in all their forms at different scale levels of matter.

In CTG uses the metric theory of relativity [7], the essence of which is dependence of metric on not only the properties of motion of system, but also on the type of test bodies, which can be both substance particles and quanta of field. Test bodies are needed to determine the metric of system in nature, for measurement procedure of scale and time, and have different properties due to the difference in equations of motion. As a result, the coefficient β in (34) may be different for different systems and should be found separately for complete definition of metric tensor. In particular, β was found in several situations, such as calculating deviation of test body's motion under the action of gravitation, and calculations of perihelion shift. Notes on the last four terms in (34) and the term with constant χ will be made later in the section "Tensors of energy".

8 The equations of motion of substance and field

To obtain the equations of motion of substance it is necessary to select in the full variation of action (2) those terms which contain variations of coordinates ξ^β. Due to the arbitrariness of ξ^β the sum of all such terms should be equal to zero. From the sum of (21), (23) and (30) we obtain:

$$\frac{1}{c} J^\sigma \nabla_\sigma u_\beta - \frac{1}{c} \Phi_{\beta\sigma} J^\sigma - \frac{1}{c} F_{\beta\sigma} j^\sigma = 0.$$

Taking into account that $J^\sigma = \rho_0 u^\sigma$, $j^\sigma = \rho_{0q} u^\sigma$, and using the operator of proper-time-derivative [7]: $u^\sigma \nabla_\sigma = \frac{D}{D\tau}$, where D denotes covariant differential, τ is proper time, the last equation can be written as follows:

$$\rho_0 \frac{Du_\beta}{D\tau} = \rho_0 a_\beta = \Phi_{\beta\sigma} \rho_0 u^\sigma + F_{\beta\sigma} \rho_{0q} u^\sigma, \qquad (35)$$

where a_β – 4-acceleration with covariant index, the first term on the right is the density of gravitational force, and the last term sets electromagnetic Lorentz force for charge density ρ_{0q}.

According to (35), the contribution to 4-acceleration of substance unit makes the gravitational acceleration $\Phi_{\beta\sigma}u^{\sigma}$ and 4-acceleration in electromagnetic field $\frac{\rho_{0q}}{\rho_0}F_{\beta\sigma}u^{\sigma}$. The physical meaning of (35) consists in the fact that it determines the impact on the substance of fields when the metric tensor of the system (this means that $\delta g_{\alpha\beta}=0$ in action function), and field potentials ($\delta D_{\beta}=0$, $\delta A_{\beta}=0$ in action function) are preset.

The relation connecting the tensor of gravitational field $\Phi^{\mu\nu}$ with its source in the form of 4-vector J^{μ}, follows from (23) and (28) as the consequence of variation δD_{β} for gravitational 4-potential. Taking into account the antisymmetry of the tensor $\Phi^{\alpha\beta}=-\Phi^{\beta\alpha}$ we obtain:

$$\nabla_{\alpha}\Phi^{\alpha\beta}=-\frac{4\pi\gamma}{c^2}J^{\beta}, \quad \text{or} \quad \nabla_{\nu}\Phi^{\mu\nu}=\frac{4\pi\gamma}{c^2}J^{\mu}. \tag{36}$$

A similar relation for electromagnetic field, from expressions for variation δA_{β} of electromagnetic 4-potential in (30) and (33) has the form:

$$\nabla_{\alpha}F^{\alpha\beta}=\frac{1}{c^2\varepsilon_0}j^{\beta}, \quad \text{or} \quad \nabla_{\nu}F^{\mu\nu}=-\frac{1}{c^2\varepsilon_0}j^{\mu}=-\mu_0 j^{\mu}, \tag{37}$$

where $\mu_0=\frac{1}{c^2\varepsilon_0}$ – vacuum permeability.

Equations (36) and (37), as it is evident after obtaining them from variation of action function, are valid in case when the variation of coordinates of substance and the variation of metric are equal to zero, that is $\xi^{\beta}=0$, $\delta g_{\alpha\beta}=0$. This means that if the motion of substance and metric of system are preset, we can calculate how the substance generates the field strengths.

If we consider the definition of gravitational tensor: $\Phi_{\mu\nu}=\nabla_{\mu}D_{\nu}-\nabla_{\nu}D_{\mu}=\partial_{\mu}D_{\nu}-\partial_{\nu}D_{\mu}$, and take the covariant derivative of the

tensor, followed by a cyclic permutation of indices, then the following equation is performed identically:

$$\nabla_\sigma \Phi_{\mu\nu} + \nabla_\nu \Phi_{\sigma\mu} + \nabla_\mu \Phi_{\nu\sigma} = 0. \tag{38}$$

Another form of (38) is:

$$\varepsilon^{\alpha\beta\gamma\delta} \nabla_\gamma \Phi_{\alpha\beta} = 0,$$

where $\varepsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita symbol or totally antisymmetrical unit tensor.

Equation (38) sets the gravitational field equations without sources, so that the set of equations (36) and (38) completely determines the properties of the gravitational field.

For electromagnetic field we have the similar formula as in (38):

$$\nabla_\sigma F_{\mu\nu} + \nabla_\nu F_{\sigma\mu} + \nabla_\mu F_{\nu\sigma} = 0 \qquad \text{or} \qquad \varepsilon^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\beta} = 0. \tag{39}$$

Equations (37) and (39) are the Maxwell equations, written in four-dimensional notation.

Relation (36) can be written in another way: $\nabla^\nu \Phi_{\mu\nu} = \frac{4\pi\gamma}{c^2} J_\mu$. If we take on both sides of this equation the contravariant derivative ∇^μ, and use the definition $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu$, then due to the symmetry and changing of the order of differentiation the left side will equal to zero. This leads to the continuity equation (mass conservation), which imposes certain conditions on 4-velocity and the density of substance:

$$\nabla^\mu J_\mu = \nabla_\mu J^\mu = \nabla_\mu \left(\rho_0 u^\mu \right) = 0.$$

For conservation of the electric charge, we have a similar relation:

$$\nabla^\mu j_\mu = \nabla_\mu j^\mu = \nabla_\mu \left(\rho_{0q} u^\mu \right) = 0 .$$

If we specify a condition for 4-vector of gravitational potential D_μ, or for 4-vector of electromagnetic potential A_μ, then it provides definite relation between scalar and vector potentials. The standard approach is Lorentz gauge, which gives the following conditions:

$$\nabla^\mu D_\mu = \nabla_\mu D^\mu = 0 , \qquad \nabla^\mu A_\mu = \nabla_\mu A^\mu = 0 . \qquad (40)$$

Substituting (40) into (36) and (37) and using $\Phi^{\mu\nu} = \nabla^\mu D^\nu - \nabla^\nu D^\mu$, $F^{\mu\nu} = \nabla^\mu A^\nu - \nabla^\nu A^\mu$, while the expression $\nabla_\nu \nabla^\nu = \Box$ is D'Alembert operator, we obtain wave equations for 4-potentials in the Lorentz gauge:

$$\Box D^\mu = -\frac{4\pi\gamma}{c^2} J^\mu , \qquad \Box A^\mu = \mu_0 j^\mu .$$

9 About applicability of equations of motion in general case

As we mentioned above, the equation for the metric (34) is correct in case when the motion of substance and field potentials are completely specified. However, in most cases only the initial state of motion and initial potentials are known, later the motion of substance is determined by field and is set indirectly. How the equation (34) should be used in this case, and under what conditions? To answer this question, we assume that the motion of substance and field potentials at a short interval of time after initial time point remain unchanged. Then we can find metric tensor from (34). After that, assuming immutability of metric and the motion of substance in the second time interval, using (36) and (37) we can calculate the derivatives of tensors $\Phi^{\alpha\beta}$ and $F^{\alpha\beta}$ by coordinates. After integration of these derivatives the field strengths can be found which are part of these tensors. Since now correct tensors $\Phi^{\alpha\beta}$ and $F^{\alpha\beta}$ are known, with the help of them on the third time interval in the equation of motion (35) the acceleration of substance and its motion can be estimated, and the motion can be adjusted. In the fourth time interval the data about motion of substance from the third interval and about fields from the second interval can be used in order to evaluate the change of metric. Further calculations are

repeated in the specified order. Thus the actual motion of substance in gravitational and electromagnetic fields, and space-time metric can be found approximately by an iterative procedure by means of the above-mentioned equations for metric, motion of substance and fields.

10 Tensors of energy

Let's return to the equation for metric (34). It is known that the covariant derivative of left side of (34) is equal to zero, which is a characteristic of the Hilbert-Einstein tensor located there. Consequently, the covariant derivative of the right side of (34) must also equal to zero:

$$\nabla_\beta\left(\phi^{\alpha\beta}+U^{\alpha\beta}+W^{\alpha\beta}+c\sqrt{g_{\mu\nu}J^\mu J^\nu}\,g^{\alpha\beta}+D_\mu J^\mu\,g^{\alpha\beta}+A_\mu j^\mu\,g^{\alpha\beta}-\chi g^{\alpha\beta}\right)=0. \tag{41}$$

Taking into account the definition of stress-energy tensor of substance (20), relations $\sqrt{g_{\mu\nu}J^\mu J^\nu}=c\rho_0$, $J^\alpha=\rho_0 u^\alpha$, and using operator of the proper-time-derivative: $u^\sigma\nabla_\sigma=\dfrac{D}{D\tau}$, we can write down:

$$\begin{aligned}\nabla_\beta\phi^{\alpha\beta}&=\nabla_\beta\left(\frac{cJ^\alpha J^\beta}{\sqrt{g_{\mu\nu}J^\mu J^\nu}}\right)=\frac{1}{\rho_0}\nabla_\beta\left(J^\alpha J^\beta\right)+J^\alpha J^\beta\nabla_\beta\left(\frac{1}{\rho_0}\right)=\\&=u^\beta\nabla_\beta J^\alpha-u^\alpha u^\beta\nabla_\beta\rho_0=\frac{DJ^\alpha}{D\tau}-u^\alpha\frac{D\rho_0}{D\tau}=\rho_0\frac{Du^\alpha}{D\tau}=\rho_0 a^\alpha,\end{aligned} \tag{42}$$

where a^α – 4-acceleration.

Now we find the covariant derivative of stress-energy tensor of gravitational field. Since the metric tensor under covariant differentiation acts as a constant, using (36), we obtain from (26):

$$\frac{4\pi\gamma}{c^2}\nabla_\beta U^{\alpha\beta} = \nabla_\beta\left(g^{\alpha\nu}\Phi_{\kappa\nu}\Phi^{\kappa\beta} - \frac{1}{4}g^{\alpha\beta}\Phi_{\mu\nu}\Phi^{\mu\nu}\right) =$$

$$= g^{\alpha\nu}\Phi^{\kappa\beta}\nabla_\beta\Phi_{\kappa\nu} + g^{\alpha\nu}\Phi_{\kappa\nu}\nabla_\beta\Phi^{\kappa\beta} - \frac{1}{4}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_\beta\Phi_{\mu\nu} - \frac{1}{4}g^{\alpha\beta}\Phi_{\mu\nu}\nabla_\beta\Phi^{\mu\nu} =$$

$$= -\Phi^{\kappa\beta}\nabla_\beta\Phi^\alpha{}_\kappa + g^{\alpha\nu}\Phi_{\kappa\nu}\frac{4\pi\gamma}{c^2}J^\kappa - \frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_\beta\Phi_{\mu\nu}. \tag{43}$$

We apply (38) to the last term in (43), on condition that tensor $\Phi_{\alpha\beta}$ is antisymmetrical:

$$-\frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_\beta\Phi_{\mu\nu} = -\frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\left(-\nabla_\nu\Phi_{\beta\mu} - \nabla_\mu\Phi_{\nu\beta}\right) =$$

$$= \frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_\nu\Phi_{\beta\mu} + \frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_\mu\Phi_{\nu\beta} = \frac{1}{2}\Phi^{\mu\nu}\nabla_\nu\Phi^\alpha{}_\mu - \frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_\mu\Phi^\alpha{}_\nu =$$

$$= \Phi^{\mu\nu}\nabla_\nu\Phi^\alpha{}_\mu.$$

Substituting this into (43) we obtain the relation between the covariant derivative of stress-energy tensor of gravitational field and 4-vector density of gravitational force:

$$\nabla_\beta U^{\alpha\beta} = -\Phi^\alpha{}_\kappa J^\kappa. \tag{44}$$

We obtain a similar expression for the covariant derivative of stress-energy tensor of electromagnetic field and 4-vector density of electromagnetic force (Lorentz force density) with (32) and (39):

$$\nabla_\beta W^{\alpha\beta} = -F^\alpha{}_\kappa j^\kappa. \tag{45}$$

Substituting (42), (44) and (45) into (41) we obtain:

$$\nabla_\beta\left(\phi^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta}\right) = \rho_0 a^\alpha - \Phi^\alpha{}_\kappa J^\kappa - F^\alpha{}_\kappa j^\kappa = 0. \tag{46}$$

The zero right side of (46) follows from the equations of substance motion in gravitational and electromagnetic fields (35). Consequently, the covariant derivative for the remaining terms in (41) must also equal to zero:

$$\begin{aligned}&\nabla_\beta g^{\alpha\beta}\left(c\sqrt{g_{\mu\nu}J^\mu J^\nu}+D_\mu J^\mu+A_\mu j^\mu-\chi\right)=\\&=\nabla^\alpha\left(c\sqrt{g_{\mu\nu}J^\mu J^\nu}+D_\mu J^\mu+A_\mu j^\mu-\chi\right)=0.\end{aligned}\tag{47}$$

In brackets in (47) there is a scalar quantity, in this case the covariant derivative ∇^α is equal to partial derivative ∂^α (that is 4-gradient). Relation (47) is automatically satisfied if we assume that the constant in brackets is set equal to zero. This gives the relation:

$$c\sqrt{g_{\mu\nu}J^\mu J^\nu}+D_\mu J^\mu+A_\mu j^\mu=\chi=const\,.\tag{48}$$

Equality (48) is necessary to perform in (34) the limit relations for tensors at infinity, where there is neither substance nor fields. As it is indicated in [4], at infinity the right side of (34) with energy tensors is equal to zero, and the space-time becomes flat, bringing to zero the left side of Hilbert-Einstein tensor. Taking into account (48) the equations for metric obtain the simplest form:

$$R^{\alpha\beta}-\frac{1}{2}g^{\alpha\beta}R=\frac{8\pi\gamma\beta}{c^4}\left(\phi^{\alpha\beta}+U^{\alpha\beta}+W^{\alpha\beta}\right).\tag{49}$$

Let's use (15) and reveal in (48) the scalar product of 4-vectors, with the help of $D_\mu=\left(\frac{\psi}{c},-\boldsymbol{D}\right)$, $A_\mu=\left(\frac{\varphi}{c},-\boldsymbol{A}\right)$, $J^\mu=\rho_0 u^\mu$, $j^\mu=\rho_{0q}u^\mu$:

$$\rho_0 c^2+\frac{\rho_0\psi u^0}{c}-\rho_0\boldsymbol{D}\cdot\boldsymbol{u}+\frac{\rho_{0q}\varphi u^0}{c}-\rho_{0q}\boldsymbol{A}\cdot\boldsymbol{u}=\chi\,.\tag{50}$$

Here $\boldsymbol{u}$ is a 3-vector, which is part of 4-velocity u^{μ}. In uncurved space-time according to special relativity theory $u^{\mu}=\left(\frac{c}{\sqrt{1-v^2/c^2}},\ \frac{\boldsymbol{v}}{\sqrt{1-v^2/c^2}}\right)$, then $u^0=\frac{c}{\sqrt{1-v^2/c^2}}$, $\boldsymbol{u}=\frac{\boldsymbol{v}}{\sqrt{1-v^2/c^2}}$, where $\boldsymbol{v}$ is the velocity of substance motion. This shows that $\frac{\rho_0\psi u^0}{c}$ is the energy density for substance in gravitational field with scalar potential ψ. The vector potential $\boldsymbol{D}$ of gravitational field is also associated with energy, but its value $-\rho_0\boldsymbol{D}\cdot\boldsymbol{u}$ can have different sign depending on the direction of vector $\boldsymbol{u}$, which is proportional to speed $\boldsymbol{v}$, and the direction of vector $\boldsymbol{D}$. The same is true in respect of the density of electromagnetic energy – it depends on the charge density ρ_{0q}, the scalar electric potential φ and vector potential $\boldsymbol{A}$ of electromagnetic field.

Now suppose that in (50) the macroscopic gravitational and electromagnetic fields are off and their potentials are equal to zero. In this case, the density of substance must reach a certain value ρ_0', which depends only on fundamental microscopic fields acting at the level of elementary particles. Then we will obtain $\chi=\rho_0'c^2$, and (50) can be rewritten as follows:

$$\rho_0c^2=\rho_0'c^2-\frac{\rho_0\psi u^0}{c}+\rho_0\boldsymbol{D}\cdot\boldsymbol{u}-\frac{\rho_{0q}\varphi u^0}{c}+\rho_{0q}\boldsymbol{A}\cdot\boldsymbol{u}=\rho_0'c^2-\varepsilon_g-\varepsilon_e, \qquad (51)$$

where ε_g and ε_e denote the energy density of substance in gravitational and electromagnetic fields, respectively.

As ε_g for substance is usually negative (due to the negative gravitational potential ψ), then from (51) it follows that density of substance ρ_0 in gravitational field becomes greater than density of substance ρ_0' in the absence of field (when the substance of a body is divided into parts and separated to infinity). The same thing can be said about mass – in gravitational field it is expected to increase due to the contribution of gravitational mass-energy of

substance in the field. Thus we have obtained the result similar to that which we have proved in [6] and [10], but in relation to contribution of mass-energy of field to the total mass of system of substance and field. Then we found that the mass of a spherical body grows due to its field, and with constant volume, this means an increase in the effective density of the substance.

We can integrate (51) over the volume of substance of a spherical uncharged body in static position, when the body is at rest and does not rotate. If the substance is infinitely slowly superimposed on the body by parts in the form of thin spherical shells with the same density of substance, we can assume that in (49) $u^0 = c$, as well as:

$$\int \rho_0 c^2 dx^1 dx^2 dx^3 = mc^2, \qquad \int \rho_0' c^2 dx^1 dx^2 dx^3 = m'c^2,$$

$$\int \rho_0 \psi dx^1 dx^2 dx^3 = -\int \frac{\gamma \rho_0 m(r)}{r} dx^1 dx^2 dx^3,$$

where m – the observed mass of the body with its radius R,

m' – the mass of substance of the body without taking into account the energy of gravitation,

$m(r) = \dfrac{4\pi r^3 \rho_0}{3}$ – the mass inside the radius r, increasing from 0 to the radius of the body R with increasing of mass.

As a result, (51) becomes equality for the masses:

$$mc^2 = m'c^2 + \int \frac{\gamma \rho_0 m(r)}{r} dx^1 dx^2 dx^3 = m'c^2 + \frac{3\gamma m^2}{5R}, \qquad (52)$$

where γ is gravitational constant.

In (52) the module of mass-energy of gravitational field is added to the mass-energy of the body. In reality, during the formation of space objects in gravitational field the virial theorem applies according to which approximately half of gravitational field energy leaves the system in the form of radiation, and

the other half heats the substance. This reduces by half the additive to mass-energy in (52).

For the main objects of stellar level of matter the contribution to (51) of energy density of substance in electromagnetic field ε_e is little in comparison to ε_g. In particular, for neutron stars, gravitational energy is equal to $E_\gamma = -\frac{k\gamma M_s^2}{R_s} \approx -2.4 \cdot 10^{46}$ J, here $k \approx 0.6$ in approximation of uniform density of substance, γ – gravitational constant, $M_s = 2.7 \cdot 10^{30}$ kg, $R_s = 12$ km – mass and radius of a typical neutron star. The electromagnetic energy reaches a maximum in magnetars at magnetic pole of which the magnetic field can be about $B_m = 1.9 \cdot 10^{11}$ T. Since magnetic energy density is given in form $\frac{B^2}{2\mu_0}$, then the integral over the entire volume inside the star and beyond it gives the magnitude of the magnetic energy of about 10^{41} J, which is considerably less than the modulus of gravitational energy.

The similar situation exists at the level of elementary particles, where according to the theory of infinite nesting of matter [8], an analog of a neutron star is a nucleon. The energy of a proton in its own field of strong gravitation is estimated with the formula $E_\Gamma = -\frac{k\Gamma M_p^2}{R_p}$, where M_p and R_p denote mass and radius of a proton, $\Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e} = 1.514 \cdot 10^{29}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$ – strong gravitational constant, e – elementary charge, ε_0 – vacuum permittivity, M_e – electron mass. The expression for electric energy of a proton in the case of uniform charge distribution is as follows: $E_e = \frac{ke^2}{4\pi\varepsilon_0 R_p}$. Consequently, for a proton the ratio of module energy of strong gravitation to electrical energy is equal to the ratio of a proton to an electron mass, and the energy of strong gravitation dominates.

The global dominance of gravitational forces over electromagnetic forces leads to possibility of formation of elementary particles of substance, massive bodies and other objects found in space. For the observable universe

cosmological constant Λ is estimated by 10^{-52} m^{-2}, and constant χ reaches $\chi \approx 5 \cdot 10^{-10}$ J/m^3. We consider that $\chi = \rho_0' c^2$ characterizes the visible universe as a whole, setting the rest energy density of substance distributed in space, without taking into account the energy fields. We further assume that gravitational fields are a consequence of flows of gravitons, which are produced by tiny particles of all the substance that exists in the universe. The more substance is in the universe, the greater is the density of substance and the more is the density of gravitons' flows. Then relation (50) supports the Einstein's idea that the inertia of a body must increase near other gravitational masses [11], which is in turn the development of Mach's principle of the impact of distant masses to acceleration of bodies.

We should remind that in the usual interpretation the cosmological constant is proportional to vacuum energy density, and still it is not known exactly what particles or fields are responsible for this energy. There are also suggestions that the cosmological constant depends on the time, on the scalar curvature of space-time (in f(R) gravity) or on the energy-momentum tensor [12].

We will further evaluate the scalar curvature R for our universe. To find R we multiply (49) by $g_{\alpha\beta}$ and taking into account the fact that $g_{\alpha\beta} R^{\alpha\beta} = R$, $g_{\alpha\beta} g^{\alpha\beta} = 4$, while for stress-energy tensors (26) and (32) there are equations: $g_{\alpha\beta} U^{\alpha\beta} = 0$, $g_{\alpha\beta} W^{\alpha\beta} = 0$. Using the expression (20), we obtain:

$$R = -\frac{8\pi\gamma\beta\rho_0}{c^2}.$$

If we assume that the density of substance can be calculated as follows $\rho_0 \approx \rho_0' = \frac{\chi}{c^2} = \frac{\Lambda c^2}{8\pi\gamma\beta}$, the curvature of cosmological constant accurate to the sign equals to: $R = -\Lambda$. At the first approximation the observed universe can be regarded as the space-time of constant negative curvature, for which the equation is $R = -\frac{12}{r^2}$. Hence the radius of such a world would be $r = \frac{\sqrt{3}\,c}{\sqrt{2\pi\gamma\beta\rho_0}} = \sqrt{\frac{12}{\Lambda}} \approx 3.4 \cdot 10^{26}$ m. On the other hand, if the observable

universe is close to the state of Schwarzschild's black hole, its radius should equal to: $r_{bh} = \frac{2\gamma M}{c^2} = \frac{\sqrt{3}\,c}{\sqrt{8\pi\gamma\rho_0}} = \frac{r}{2}$. In both cases, in determination of space-time curvature, and in evaluating the radius of the black hole, speed of light is used as a measure of speed and means for measuring.

11 Analysis of equation of motion in relation to mass

According to (46) the equation of substance motion in gravitational and electromagnetic fields has the form:

$$\nabla_\beta \left(\phi^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta} \right) = 0 . \tag{53}$$

Let's consider (53) in weak field limit, where the special relativity theory is valid and the covariant derivative becomes the 4-gradient. In this case we obtain:

$$J^\mu = \rho_0 u^\mu = \rho_0 \left(\frac{c}{\sqrt{1 - v^2/c^2}}, \quad \frac{\boldsymbol{v}}{\sqrt{1 - v^2/c^2}} \right),$$

and for the scalar components of stress-energy tensor of substance (20) we can write down:

$$\phi^{00} = \frac{\rho_0 c^2}{1 - v^2/c^2} , \qquad \phi^{0i} = \frac{K^i}{c} = \frac{\rho_0 c v^i}{1 - v^2/c^2} , \tag{54}$$

where the index $i = 1, 2, 3$, so that in the Cartesian coordinate system $v^1 = v_x$, $v^2 = v_y$, $v^3 = v_z$, and vector $\boldsymbol{K} = \frac{\rho_0 c^2 \boldsymbol{v}}{1 - v^2/c^2}$ is 3-vector of mechanical energy flow density or relativistic Umov's vector.

The gravitational tensor $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$ is defined by the 4-potential $D_\mu = \left(\frac{\psi}{c}, -\boldsymbol{D}\right)$ of the gravitational field, which depends on the scalar potential ψ and vector potential $\boldsymbol{D}$ of the gravitational field. We shall introduce gravitational acceleration $\boldsymbol{G}$ and torsion field $\boldsymbol{\Omega}$ (gravitomagnetic field) as follows:

$$\boldsymbol{G} = -\nabla\psi - \frac{\partial \boldsymbol{D}}{\partial t}, \qquad \boldsymbol{\Omega} = \nabla \times \boldsymbol{D}.$$

With the help of the quantities $\boldsymbol{G}$ and $\boldsymbol{\Omega}$ the scalar components of the stress-energy tensor of gravitational field (26) can be represented as follows:

$$U^{00} = -\frac{1}{8\pi\gamma}(G^2 + c^2\Omega^2), \qquad U^{0i} = \frac{H^i}{c}, \tag{55}$$

where H^i denotes the components of 3-vector energy flow density of gravitational field (Heaviside vector) $\boldsymbol{H} = -\frac{c^2}{4\pi\gamma}[\boldsymbol{G} \times \boldsymbol{\Omega}]$.

For the stress-energy tensor of electromagnetic field (32) in substance which is not magnetized and not polarized in the limit of special relativity we obtain similarly:

$$W^{00} = \frac{\varepsilon_0}{2}(E^2 + c^2B^2), \qquad W^{0i} = \frac{P^i}{c}, \tag{56}$$

where P^i denote the components of 3-vector of electromagnetic energy flow density (Poynting vector) $\boldsymbol{P} = \varepsilon_0 c^2 [\boldsymbol{E} \times \boldsymbol{B}]$.

Substituting (54), (55) and (56) into (53) for $\alpha = 0$ and for the replacement ∇_β on ∂_β in Minkowski space makes an expression of Poynting's theorem:

$$\partial_\beta\left(\phi^{0\beta}+U^{0\beta}+W^{0\beta}\right)=0, \quad \text{or} \quad \frac{\partial}{\partial t}\left(\phi^{00}+U^{00}+W^{00}\right)=-\nabla\cdot(\boldsymbol{K}+\boldsymbol{H}+\boldsymbol{P}). \tag{57}$$

The values ϕ^{00}, U^{00} and W^{00} set the energy density of substance, gravitational and electromagnetic fields respectively.

Let's integrate (57) over the entire volume of space occupied by substance and field. We shall consider the quasi-stationary case, when all energy flows are closed so that the volume integral of divergence of the right side of (57) becomes close to zero. This means that the amount of energy flows, extending forever into infinity is little. Then taking into account (54), (55) and (56) we should obtain:

$$\int\left[\frac{\rho_0 c^2}{1-v^2/c^2}-\frac{1}{8\pi\gamma}(G^2+c^2\Omega^2)+\frac{\varepsilon_0}{2}(E^2+c^2B^2)\right]dV=const=m'c^2, \tag{58}$$

$$\int\frac{\rho_0 c^2}{1-v^2/c^2}dV=\int\frac{\rho_0 c^2}{1-v^2/c^2}\sqrt{1-v^2/c^2}\,dV_0=\int\frac{c^2 dm}{\sqrt{1-v^2/c^2}}=mc^2+E_k,$$

$$mc^2=m'c^2-E_k+\int\left[\frac{1}{8\pi\gamma}(G^2+c^2\Omega^2)-\frac{\varepsilon_0}{2}(E^2+c^2B^2)\right]dV.$$

We denoted by m' the mass of substance when the substance is separated to infinity and is in rest there. In this case all the fields G, Ω, E and B will tend to zero. Equation (58) shows that the mass of substance m in relation to the mass of substance m' increases due to the contribution of total mass-energy of gravitational field and decreases due to the contribution of mass-energy of electromagnetic field and the kinetic energy of motion of substance E_k. Actually (58) confirms the relation (52) from the previous section.

12 Additional notes

In our view, the equation of motion of substance (35) should look a little differently:

$$\frac{DJ_\beta}{D\tau}=\frac{D(\rho_0 u_\beta)}{D\tau}=\Phi_{\beta\sigma}\rho_0 u^\sigma+F_{\beta\sigma}\rho_{0q}u^\sigma. \qquad (59)$$

In (59) the substance density ρ_0 has been included under the sign of total derivative with respect to proper time. This allows us to describe cases where the substance density changes and thus creates an additional acceleration of substance. Meanwhile, equation (35) was obtained from the variation of coordinates described in [2] and [4], with constant mass in the variation. This led to the fact that (35) differs from (59), as in case $\rho_0=const$, and therefore ρ_0 can be outside of the total differential.

It is interesting that we can choose the stress-energy tensor of substance so that its covariant derivative just gives the rate of change of substance's 4-current. This tensor has an unusual form in terms of indices, but formal covariant derivative gives the correct result. Instead of (20) we shall write down: $\phi^{\alpha\beta}=\frac{2cJ^\alpha J^\beta}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}$. Taking into account the continuity equation $\nabla_\beta J^\beta=0$ we obtain:

$$\nabla_\beta\phi^{\alpha\beta}=\nabla_\beta\left(\frac{2cJ^\alpha J^\beta}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}\right)=\frac{2cJ^\beta\nabla_\beta J^\alpha}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}+2cJ^\alpha J^\beta\nabla_\beta\left(\frac{1}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}\right)=$$

$$=\frac{2cJ^\beta\nabla_\beta J^\alpha}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}-\frac{cJ^\alpha J^\beta}{(g_{\alpha\beta}J^\alpha J^\beta)^{1.5}}\nabla_\beta\left(g_{\alpha\beta}J^\alpha J^\beta\right)=\frac{cJ^\beta\nabla_\beta J^\alpha}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}=u^\beta\nabla_\beta J^\alpha=\frac{DJ^\alpha}{D\tau}.$$

In fact, we have found the tensor $\phi^{\alpha\beta}=\frac{2cJ^\alpha J^\beta}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}$ in (17). The reason that we have not used it as it is described here, is that instead of the exact form of variation δJ^β as the function of coordinate's variations, the simplified form (18) was used in derivation of motion equations. This leads to (35), but not to (59).

From variation (48) it follows that the total variation of mass 4-current δJ^{β} is associated with variation of metric tensor $\delta g_{\mu\nu}$, variation of electromagnetic 4-current δj^{μ} and variations of 4-potentials:

$$\delta\left(c\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}+D_{\mu}J^{\mu}+A_{\mu}j^{\mu}-\chi\right)=0, \tag{60}$$

$$\frac{1}{2}u^{\mu}J^{\nu}\delta g_{\mu\nu}+u_{\nu}\delta J^{\nu}+D_{\mu}\delta J^{\mu}+J^{\mu}\delta D_{\mu}+A_{\mu}\delta j^{\mu}+j^{\mu}\delta A_{\mu}=0.$$

The relationship of variations in (60) is connected through variation of coordinates ξ^{β} in such a way that the mass of any local volume does not change during variation. However, we can admit a situation when the mass-energy is converted into radiation energy, or substance density changes due to inflow or outflow of mass. Then, some results presented in this paper will require a corresponding change.

Let's now substitute (48) into (2), and with equations $\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}=c\rho_0$, $-2k\Lambda-\frac{\chi}{c}=0$, we obtain:

$$\delta S=\delta\int\left(kR+\frac{c}{16\pi\gamma}\Phi_{\mu\nu}\Phi^{\mu\nu}-\frac{c\varepsilon_0}{4}F_{\mu\nu}F^{\mu\nu}\right)\sqrt{-g}\,d\Sigma=0.$$

After implementation of variation in this equation we will obtain the equations for metric (49), but without the tensor $\phi^{\alpha\beta}$:

$$R^{\alpha\beta}-\frac{1}{2}g^{\alpha\beta}R=\frac{8\pi\gamma\beta}{c^4}\left(U^{\alpha\beta}+W^{\alpha\beta}\right). \tag{61}$$

The metric obtained in (61) is as it should be outside the substance, and this metric depends directly on the magnitude of existing fields and indirectly on the distribution of substance in this field.

In CTG there is a concept of geodesic line, which coincides with the expression of general relativity theory, but only for field quanta. The equation of motion (35), taking into account $D\tau = d\tau$, can be written as:

$$\frac{D}{d\tau}\left(\frac{dx_\beta}{d\tau}\right) = \frac{Ddx_\beta}{d\tau d\tau} = \Phi_{\beta\sigma}\frac{dx^\sigma}{d\tau} + \frac{\rho_{0q}}{\rho_0}F_{\beta\sigma}\frac{dx^\sigma}{d\tau}. \qquad (62)$$

For field quanta we obtain $ds = cd\tau = 0$. Multiplying (62) by $d\tau d\tau$, we obtain the equality to zero of the right side: $Ddx_\beta = 0$. Dividing this by the square differential of time coordinate λ, which measures time along the trajectory of quantum, and recalling the definition of operator of proper-time-derivative, we obtain the geodesic equation in covariant indices:

$$\frac{Ddx_\beta}{d\lambda d\lambda} = \frac{D}{d\lambda}\left(\frac{dx_\beta}{d\lambda}\right) = \frac{dx^\sigma}{d\lambda}\nabla_\sigma\left(\frac{dx_\beta}{d\lambda}\right) = \frac{dx^\sigma}{d\lambda}\partial_\sigma\left(\frac{dx_\beta}{d\lambda}\right) - \Gamma^\gamma_{\sigma\beta}\frac{dx^\sigma}{d\lambda}\frac{dx_\gamma}{d\lambda} =$$
$$= \frac{d}{d\lambda}\left(\frac{dx_\beta}{d\lambda}\right) - \Gamma^\gamma_{\sigma\beta}\frac{dx^\sigma}{d\lambda}\frac{dx_\gamma}{d\lambda} = 0. \qquad (63)$$

While the field quanta are distributed outside the substance in a given field (gravitational and electromagnetic), their movement takes place in accordance with the equation of motion (63), and the space-time metric is determined from (61). What can change while the field quanta pass through the substance? If the substance is rare and does not interact with quanta, the quanta move between the particles of substance. Then in principle, equation (61) should be valid for the metric with the amendment that it is now necessary to take into account the stress-energy tensor of field of strong gravitation acting at the level of elementary particles. This new tensor should look as a supplement to the stress-energy tensor $U^{\alpha\beta}$ of normal gravitation (26), replacing the constant of gravitation γ by the constant of strong gravitation Γ, and with a factor of proportionality. In practice instead of using this new tensor, it is replaced by the stress-energy tensor of substance $\phi^{\alpha\beta}$, and one say that "the substance alters the space-time metric inside, and affects the metric beyond its borders".

This leads to the equation for the metric in the form of (49). But another interpretation is possible – a field always affects the metric, whereas the role of substance is reduced only to creation of field. In this case it is necessary to impose the condition on the properties of test objects with the help of which we study metric and find metric tensor components – these test bodies must interact with the substance at a distance and only through the fields, without mechanical contact randomly changing the movement.

The above interpretation of relationship between substance, metric and field is difficult in general relativity, in which gravitation is hiding in the shadow of geometrical metric field and losing its physical essence. Metric gravitational field (metric of space-time) in general relativity depends on the substance and electromagnetic field and is fully determined by them. But how does the substance change physically the metric field even if it is far away from it? What is the relationship mechanism between the substance and the field? All this remains a mystery.

In covariant theory of gravitation the Fatio-Le Sage's gravitation theory is considered as the basic idea of generating gravitational field, which allows to describe in the same way strong gravitation at the level of elementary particles and usual macroscopic gravitation [13], as well as electromagnetic interaction between bodies [7]. Quanta of gravitation, which are formed by relativistic objects at the lower levels of matter presumably in the form of electromagnetic radiation and neutrinos, become gravitons for the objects of higher levels of matter and create there gravitational interaction. The gradients of energy density of gravitons' flows may be considered as gravitational field strengths. Then the gravitational potential is the difference between the energy density of gravitons' flows near or inside the bodies, and the energy density of gravitons' flows at infinity in the absence of bodies. These flows of gravitons are responsible for deviation of test particles and field quanta near the massive bodies. In this picture the field quanta of lower levels of matter generate macroscopic fields and form macroscopic metric, and the substance (regarded as an aggregation of objects from different levels of matter, distinguishing by their characteristic sizes and masses) interacts with the field quanta and generates them.

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AN ALTERNATIVE MODEL FOR THE ROTATION OF SPIRAL GALAXIES: THE CONNECTIONS AMONG SHAPE, KINEMATICS AND EVOLUTION

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Abstract

It is proposed that the arms of spiral galaxies are formed by the continuous outflow of matter from their centers. It is then shown that the ratio between the radial and tangential velocities of the outflow is the parameter responsible for the logarithmic spiral structure of spiral galaxies. The fitting of some spiral galaxies to the model allows the calculation of the radial velocities of matter in these galaxies and such values completely agree with the observational data. An approximate general equation is proposed for the description of the arms of spiral galaxies with or without bars. Some important consequences are discussed with respect to dark matter, galactic evolution and cosmology. It is concluded that a quantitative representation of the dynamics of the spiral galaxies can be entirely represented with data on their formation without any need to use the far-reaching conjecture of dark matter. We finally indicate that this conclusion is fully in agreement with the absence of dark matter inferred from Santilli's IsoRedShift within the inhomogeneous and anisotropic inner galactic medium that light has to traverse before reaching intergalactic spaces, as well as with the recent demise of symmetries and other recent advances.

Keywords: Rotation models of spiral galaxies; Rotation of spiral galaxies; Spiral galaxies; Spiral structure; Galactic evolution; Santilli IsoRedShift; Dark matter.

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Introduction

The current status quo for the rotation of spiral galaxies is still based on the density wave theory which states that the matter of the disk becomes distributed in spiral arms due to the action of a wave-like perturbation in the form of quasi-steady global modes of the disk ([1],[2],[3]). These three references are just examples of a long list of proposals within the same general framework. Other approaches defend that the spiral structure is a short-lived, transient phenomenon triggered by gravitational instabilities. Two of many references in this line are Goldreich & Lynden-Bell [4], and Julian & Toomre [5].

A quite simple argumentation against the density wave theory is that according to this theory we would have to have many spirals with 3, 4, and even 5 arms. Observations, however, have shown that almost all spirals have only two arms.

A very recent study by Foyle et al. [6] based on observations of 12 spiral galaxies discard the density wave theory in its simplest form as being an "important aspect of explaining spirals in large disk galaxies", but the authors wrongly conclude that the spiral structure is not a long-lived phenomenon.

We present an alternative model for the description of spiral galaxies based on the outflow of matter from their cores and show that the spiral structure is inherent to the existence of the galaxy. Outflows of matter from the centers of galaxies have been reported since a long time ago. Let us present some examples. Very recent data [7] of NGC 6240, which is considered a typical protogalaxy show that "approximately 70% of the total radio power at 20cm originates from the nuclear region ($\leq 1.5\,kpc$), of which half is emitted by two unresolved ($R \leq 30\,pc$) cores and half by a diffuse component. Nearly all of the other 30% of the total radio power comes from an arm-like region extending westward from the nuclear region". NGC 2992 presents a jet-like structure and a circum-nuclear ring [8]. Falcke and Biermann [9] report that there is a large scale emission-like jet going outward from the core of NGC 4258 with a mass of about 4×10^{35} kg and with a kinetic power of approximately 10^{42} ergs/s and expansion velocity of about 2000km/s. Brunthaler et al. [10] report the first superluminal jet with a velocity of about 1.25c in the Seyfert spiral galaxy III Zw 2. For superluminal as well as subluminal speeds within physical media verifying causality laws in view of the universal Lorentz-Poincaré-Santilli (LPS) isosymmetry for interior dynamical problems, one can see Ref. [11], and the comments in the Concluding Remarks.

Balmaverde and Capetti [12] have reported in 2006 that "Considering the radio structure, several objects of our CoreG sample have a radio-morphology with well developed jets and lobes: UGC 7360, UGC 7494 and UGC 7654 are FR I radio-galaxies part of the 3C sample (3C 270, 3C 272.1 and 3C 274), while in the Southern sample we have the well studied radio-galaxies NGC 1316 (Fornax A), a FR II source, NGC 5128 (Cen A) and IC 4296. A literature search shows that at least another 11

sources have extended radio-structures indicative of a collimated outflow, although in several cases this can only be seen in high resolution VLBI images, such as the mass scale double-lobes in UGC 7760 or the one-sided jet of UGC 7386". Sturm et al. [13] have just reported massive molecular outflows from the centers of ultraluminous infrared galaxies (ULIRGs). As the authors state the terminal velocities in some of these outflows exceed 1000 km/s "and their outflow rates (up to ~ 1200 solar masses per year) are several times larger than their star formation rates". Middleberg et al. [14] report radio observation of Seyfert galaxies NGC 7674, NGC 5506, NGC 2110 and Mrk 1210, and conclude that "Our results confirm and extend earlier work showing that the outward motion of radio components in Seyfert galaxies is non-relativistic on pc scale. We briefly discuss whether this non-relativistic motion is intrinsic to the jet-formation process ...." Muñoz-Tuñon and Beckman [15] analyze the consequences of mass outflows in the circumnuclear zones of galaxies. They have found "in addition to a ring structure in the gas, there is often measurable expansion with higher radial velocities occurring near the nucleus" and also they show that "radially progressive bursts of star formation can account for a wide range of these observed phenomena and could be related to the presence of *liners* in the interstellar medium close to the nucleus." Last April Alatalo et al. [16] have reported the discovery of an AGN-Driven Molecular Outflow in the early-type galaxy NGC 1266 which is classified as an S0 without arms. The molecular outflow has a molecular mass of 2.4×10^7 Msuns. As the authors observe "The star formation in NGC 1266 is insufficient to drive the outflow, and thus it is likely driven by the active galactic nucleus (AGN)". A very important work that shows the initial formation of spiral arms in a galaxy is the paper of Stark et al. [17] entitled The formation and assembly of a typical star-forming galaxy at redshift $z \approx 3$. The authors report studies of a galaxy about 2-3 Gyr after the Big Bang. One of the very significant results of the work is the regular, bi-symmetric velocity field revealed by the O III emission lines measurements. Examining the velocity field in more detail, the authors extracted a rotation curve with a circular velocity of about 67 km/s. Figure 1 of the article clearly shows an arm-like structure. Mark Swinbank who is one of the authors, commenting on the paper on the site EurekAlert! [18], has said that "The distribution of gas seen with our amazing resolution indicates we are witnessing the gradual build-up of a spiral disk with a central nuclear component". Recently, in 2001, Wilson et al. [19], using high resolution X-ray observations with Chandra, have managed to solve the puzzle concerning the anomalous ghostly opposite arms of M106 (NGC 4258) which are dominated by young stars : they are jets that originate in the nucleus of the galaxy.

Another aspect of the subject is provided by analyses of metallicity gradients in spiral galaxies. The NED/IAC/Caltech document [20] on this issue is a thorough text which takes into account the works of many researchers. The text shows that the data for the Milky Way are in line with those from other spirals, and that the metallicity data expressed in terms of $12+\log(O/H)$ decreases with the distance from the centers

of spirals, clearly showing that their disks are younger than their bulges and that the hydrogen has its origin in the centers of galaxies.

Taking a closer look at the morphologies of some galaxies we can clearly see jets/arms coming out or their nuclei. It is the case of some galaxies classified as peculiar galaxies. All pictures of UBVR images (UGC ´s and VV 114*) shown below are credited to Hibbard, Liu & Armus [21]. A very enlightening image is that of UGC 04264* (Fig 1) which shows two young arms in the lower spiral galaxy. The lower arm is not affected by tidal forces.

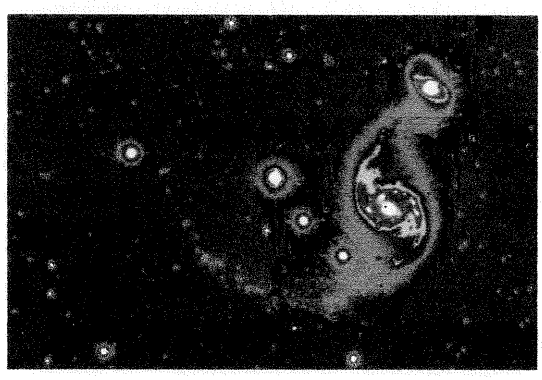

Fig.1: UBVR image of UGC 04264*. The arms in the spiral galaxy are young and the lower arm is not affected by tidal forces from the interacting galaxies.

Still another clear example is the image of UGC 06748* (Fig. 2) where we can observe young arms in the spiral galaxies being formed. Observe that the top arm of the left galaxy and the right arm of the middle galaxy do not suffer tidal forces.

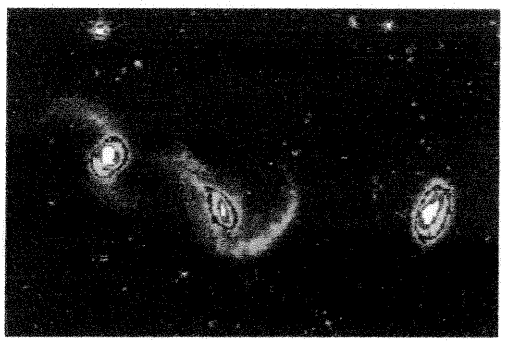

Fig. 2. UBVR image of UGC 06748*. On the left we see a pair of interacting spiral galaxies with young arms.

The remarkable image below (Fig. 3) of UGC 08929* reveals the formation of a very young arm in the spiral galaxy. The arm/jet is just beginning to curve and on the other side we already see some protuberance being formed.

Fig. 3. UBV image of a very young arm just beginning to curve. Observe the protuberance on the opposite side.

Fig. 4 below is another quite remarkable UBVR image which shows spiral arms just being formed. We clearly notice that the upper arm is much more developed than the lower arm.

It is worth mentioning that along with outflows of gases there are also inflows of gases towards the centers of galaxies. A particular inflow is actually the result of a previous outflow because if the outflow velocities are smaller than the escape velocity from the very massive center, the gases just fall back and suffer inflows. This happens a lot in barred spirals.

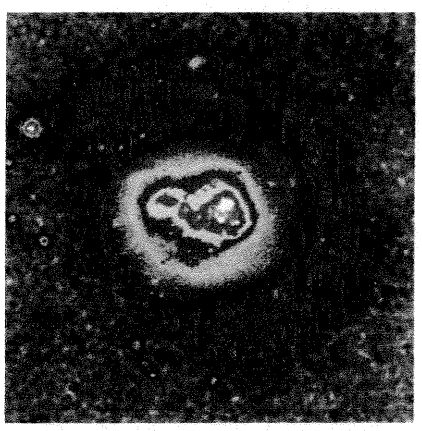

Fig. 4. Spiral arms just being formed in spiral galaxy VV 114* (Arp 236).

Closing this introduction let us address the mathematical description of the arms in spiral galaxies. It is well known that the arms of spiral galaxies are excellently described by a logarithmic spiral of the form

$$r = ae^{b\theta} \tag{1}$$

as proposed by Danvar [22]. The constants a and b are just constants which can be appropriately chosen.

2 The model

Taking into account what was shown and discussed above this work proposes that the spiral arms are formed by the shedding of matter from the nuclei of spiral galaxies. This is actually an old idea, proposed in 1964 by Oki et al. [23]. So, let us consider that a certain extended mass of gas m is ejected from the bulge of the galaxy with a radial velocity v_r as is shown in Fig. 5. In the bulge the mass m was rotating with an angular velocity Ω. When it leaves the bulge at a later time v_θ is not affected by the radial driving forces that cause the shedding of matter, and as it is shown below the mass keeps its angular momentum maintaining the tangential velocity approximately constant. Let us recall again that m is not pointlike. The Milky Way and other galaxies show that the mass m frequently has the form of an arc of matter which gets approximately distributed along a spiral so that we have an equation of the form for the angular momentum of m

$$mRv_\theta = \sum_i m_i r_i v_{\theta_i} \tag{2}$$

where R is the radius of the bulge, v_{θ_i} are the velocities of the different parts of the extended mass m which are located at r_i, just after having left the bulge. We notice that after having left the bulge the mass m can continue with an average v_θ given by

$$v_\theta = \frac{1}{mR}\sum_i m_i r_i v_{\theta_i} = \sum_i \left(\frac{m_i}{m}\right)\left(\frac{r_i}{R}\right) v_{\theta_i} \tag{3}$$

in which, since $m_i / m < 1$ and $r_i / R > 1$, v_{θ_i} can thus have values around v_θ. We obtain more detail on this if we analyze the behavior of the kinetic energy. Just before leaving the bulge the mass m(in the form or an arc, for example) has the kinetic energy

$$K = \frac{1}{2} m\left(v_r^2 + v_\theta^2\right) \tag{4}$$

and just after having left the bulge the mass m has the kinetic energy

$$K = \frac{1}{2}\sum_i m_i\left(v_{r_i}^{\,2} + v_{\theta_i}^{\,2}\right) \tag{5}$$

Disregarding the small variation of the gravitational potential, K is conserved and thus we have

$$\frac{dK}{dt} = \sum_i m_i\left(v_{r_i}\frac{dv_{r_i}}{dt} + v_{\theta_i}\frac{dv_{\theta_i}}{dt}\right) = 0 \tag{6}$$

whose solutions are

$$v_{r_i}\frac{dv_{r_i}}{dt} + v_{\theta_i}\frac{dv_{\theta_i}}{dt} = 0 \tag{7}$$

with $\frac{dv_{\theta_i}}{dt} \neq 0$, $\frac{dv_{r_i}}{dt} \neq 0$, and

$$\begin{cases} v_{r_i} = const \\ v_{\theta_i} = const \end{cases} \tag{8}$$

In the case of (7) $\frac{dv_{r_i}}{dt}$ and $\frac{dv_{\theta_i}}{dt}$ have opposite signs. The analysis of Shetty et al. [24] on streaming of matter in M51 clearly shows that v_{r_i} and v_{θ_i} vary together about their mean average values.

However, observations have shown that Nature prefers the solution given by (8), and our fittings below show that this is indeed the case. The reason for this lies in the fact that the driving forces that shed matter outward are radial forces and, hence, do no work in the direction of v_{θ_i}. The simple arguments above show that the constancy of the tangential velocity of spiral galaxies is directly

related to the ejection of matter from their bulges and to the conservation of mass, energy and angular momentum, and thus, there is no need for dark matter. Thus

$$v_\theta \approx const \tag{9}$$

throughout the galaxy.

After having left the bulge, after a time interval Δt, the center of mass of the extended mass m will be located at point P, at a distance r from the center O (Fig. 5), and since v_θ remains approximately constant, we have

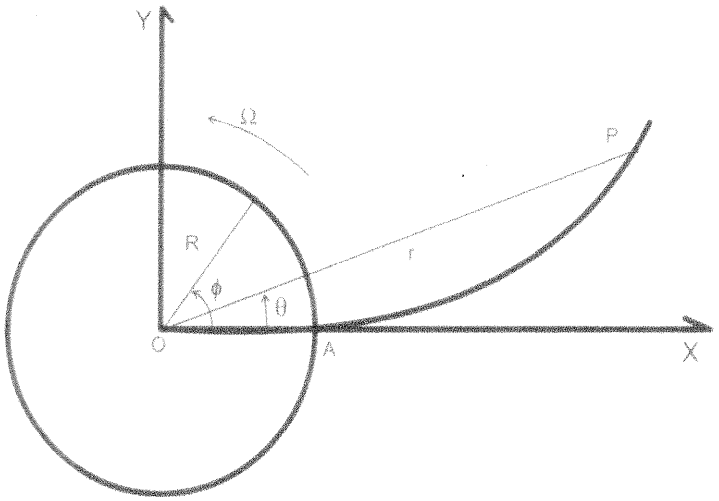

Fig. 5. While the bulge radius sweeps an angle ϕ, the mass m makes an angle θ in its displacement from A to P.

$$r\omega = r\frac{d\theta}{dt} = R\frac{d\phi}{dt} = R\Omega = v_\theta \tag{10}$$

where ϕ is the angle that the center of mass of m would have if it had not been ejected from the bulge, that is, it is the angle that the bulge made during the time interval Δt. Thus, (10) yields

$$\omega = \frac{R\Omega}{r} \tag{11}$$

From (10) we also obtain

$$d\theta = \omega dt = \frac{R\Omega}{r} dt = \frac{V_\theta}{rV_r} dr \tag{12}$$

where we have made use of the fact that $V_r = \frac{dr}{dt}$.

The first approximation is to consider the radial velocity V_r approximately constant. In this case the integral of (12) is just

$$r = Re^{\Gamma\theta} \tag{13}$$

where $\Gamma = \frac{V_r}{V_\theta}$. (13) is Danvar equation, but now we see that Γ is a very important parameter, directly related to the kinematics of the galaxy. It was deduced by de Souza quite some time ago [25,26]. Substituting (13) into (11) we obtain

$$\omega = \Omega e^{-\Gamma\theta} \tag{14}$$

and thus the arms lag the bulge exponentially with respect to θ. Since $\Omega = \frac{d\phi}{dt}$,

$$\phi = \frac{1}{\Gamma}\left(e^{\Gamma\theta} - 1\right) \tag{15}$$

This relation is important because knowing the maximum θ for a certain spiral arm we can find the maximum value of ϕ and find out how much the bulge rotated since the beginning of the formation of the spiral arms. Of course, this is very important for studying galactic evolution. We can also define the lagging angle $\Psi = \phi - \theta$ (Fig. 6).

It is important to mention that the beginnings of the formations of the two arms may occur at different times, and the arms can have different radial velocities. That is why there are asymmetric spirals with an arm much more developed than the other one. There are even spiral galaxies with a single arm such as NGC 4725.

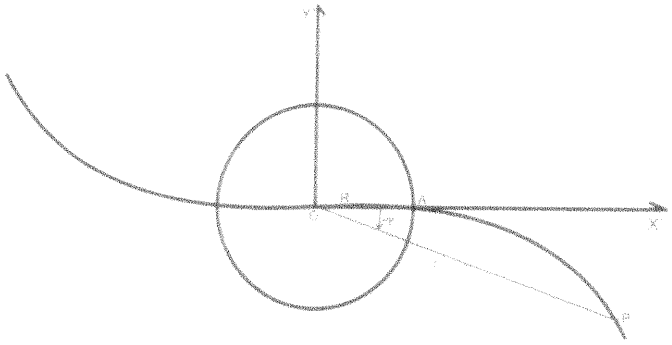

Fig. 6. The lagging angle $-\Psi = \theta - \phi$ which is measured with respect to the initial stream of matter, across a diameter of the bulge by an observer corotating with the bulge.

In the data of Ganda et. al. [27] of 18 galaxies we find galaxies where v_r increases slightly with r, although in most of them v_r decreases slightly with r. Therefore, our approximation above is quite justified. If we consider the variation in v_r with the gravitational field we should also consider other effects such as interaction between consecutive arms, and this is a very complex effect and is hard to be taken into account.

3 Application of the model to some spirals

We begin the application of (13) to the grand design galaxies M51 and M74 and to the barred spiral NGC 1300. In all galaxies below the data points were visually captured with the use of the software *plot digitalizer.* Since we are dealing with the logarithm of a fraction in the fitting of Γ, the error bar is quite small because the logarithm smoothes out errors. Thus we estimate for the fitted values of Γ an error bar of about 1-2%.

3.1 M51

We followed the black dust lane of the lower arm in Fig. 7 below (the arm that goes towards M51B). We took the lagging angle $-\Psi$ with respect to the beginning of the arm, that is, with respect to $\phi = 0$, and thus, $\theta = -\Psi$ in intervals of 30^o, and measured the corresponding values of r in terms of the bulge radius R.

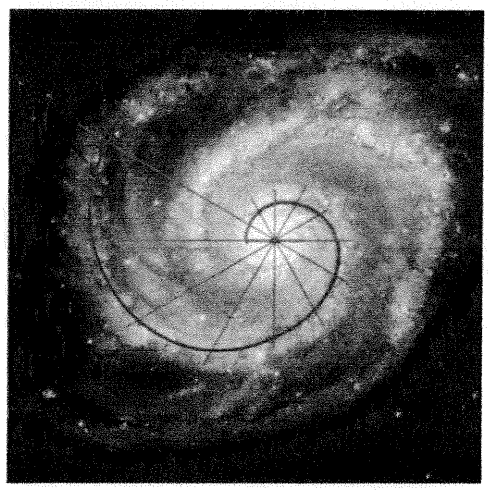

Fig. 7. The considered data points were close to the above spiral, following a dust lane (NASA/HST photo).

The obtained values for r/R and their corresponding θ and Γ are listed in Table 1. The average value of Γ is 0.37, and thus $v_r / v_\theta = 0.37$.

On p. 1151 of the paper *Kinematics of spiral-arm streaming in M51*, Shetty et al. [24] present a detailed analysis of v_r and v_θ in terms of an arm phase angle defined by the authors. It is clearly shown that v_r values vary around 50km/s and v_θ have values around 150km/s, and thus the ratio $\Gamma = \frac{v_r}{v_\theta} \approx 0.30$ is quite close to the above value of $\Gamma = 0.37$. As we will see below this is, actually, the worst discrepancy.

Table 1. The calculated values of Γ for spiral galaxy M51.

$\theta = -\Psi$	r/R	Γ
30^o	1.23	0.39
60^o	1.45	0.36
90^o	1.82	0.38
120^o	2.09	0.35
150^o	2.49	0.35
180^o	2.86	0.34
210^o	4.45	0.41
240^o	4.82	0.38

3.2 M74

In this case we chose the spiral arm shown in Fig. 8 and considered points corresponding to bright stars around the middle of the arm. We took the lagging angle as in the case of M51.

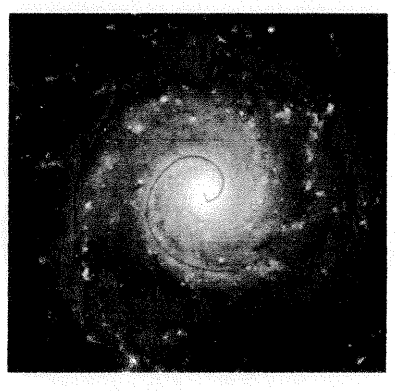

Fig. 8. The fitted spiral of NGC 628 considered for the calculation. The spiral is along the bright patch of the arm (NASA/HST photo).

Table 2. The calculated values of Γ for spiral galaxy M74.

$\theta = -\Psi$	r/R	Γ
60^o	1.38	0.31
90^o	1.75	0.36
120^o	2.25	0.39
150^o	2.63	0.37
180^o	3.13	0.36
210^o	3.63	0.35
240^o	3.88	0.32
270^o	4.25	0.31
300^o	4.63	0.29

According to Kamphuis & Briggs [28] v_θ for NGC 628 (M74) is about 200 km/s and according to Ganda et al. [27] its v_r is about 70 km/s, and thus, $\Gamma \approx \frac{70}{200} = 0.35$ which is quite close to our calculated values above (Table 2) whose average is 0.34.

3.3 NGC 1300

The considered value for R was half of the diameter between the ends of the dust lanes in the bright nucleus. Then this line was displaced to the beginning of the spiral arms for the calculation of Γ (Fig. 9). As in the previous examples, intervals of 30^o were used along the dust lane of the arm.

Table 3. The calculated values of Γ for the barred spiral galaxy NGC 1300.

$\theta = -\Psi$	r/R	Γ
60^o	1.31	0.52
90^o	1.74	0.53
120^o	2.34	0.54
150^o	3.00	0.42

The calculated values are shown in Table 3. Above 150^o Γ decreases too much, probably because, due to its shape, the arm is attracted a lot towards the bar. The average of the calculated values below is 0.50.

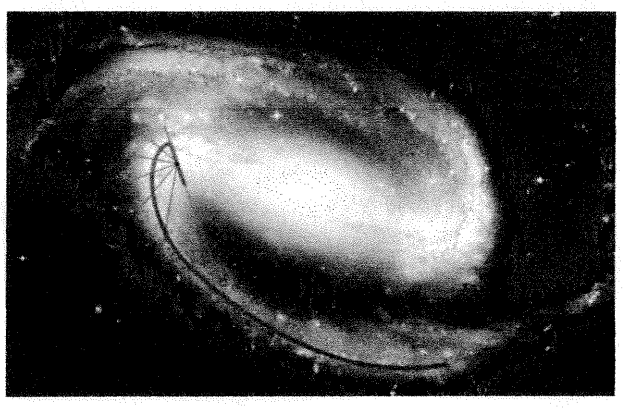

Fig. 9. We considered the dust lane of the lower arm of NGC 1300 for the calculation of Γ (NASA/HST photo).

According to Aguerri et al. [29] $v_\theta \approx 220\,\mathrm{kms}^{-1}$ and Lindblad et al. [30] report a v_r in the arms of the order of $120\mathrm{kms}^{-1}$, yielding, thus, a value of $\Gamma \approx 0{,}545$ which is very close to the calculated values above.

3.4 NGC 4030

In this case, since the bulge is very fuzzy, we considered angular differences with respect to a baseline across the center of the nucleus. Fig. 10 below shows the arm which was considered and Table 4 presents the results whose average value is 0.37.

Table 4. The calculated values of Γ for the spiral galaxy NGC 4030. r_b means the distance to the center of the nucleus across the baseline.

$\theta = -\Psi$	r / r_b	Γ
30^o	1.38	0.40
60^o	1.46	0.36
90^o	1.77	0.36
120^o	2.15	0.37

Ganda et al [27] report a value $v_r \approx 90$ km/s and Mathewson & Ford [31] say that v_θ is about 236 km/s, and, thus, $\Gamma \approx 0.38$ which is quite close to the above calculated average value.

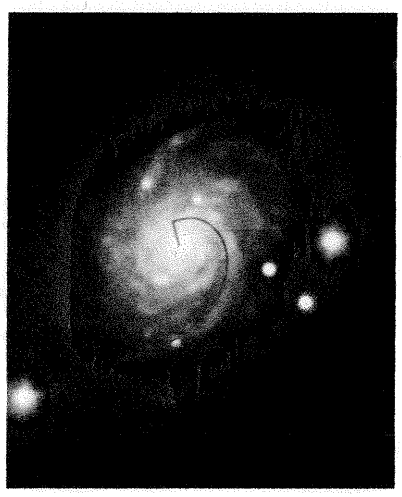

Fig. 10. The fitted arm of the spiral NGC 4030 (Penryn, California photo).

3.5 NGC 1042

We performed the fitting in the longer arm, as shown below in Fig. 11 and measured the ratios r/R in intervals of 30°. The calculated values for Γ are listed in Table 5. Their average value is about 0.69.

Table 5. The calculated values of Γ for the spiral galaxy NGC 1042.

$\theta = -\Psi$	r/R	Γ
60^o	2.0	0.66
90^o	3.0	0.70
120^o	4.6	0.73
150^o	5.6	0.66

Fig. 11. The arm of NGC 1042 considered in the fitting (NASA/HST photo).

Ganda et al. [27] report $v_r \approx 50\,\mathrm{kms}^{-1}$ and Kornreich et al. [32] present $v_\theta \approx 69.4\,\mathrm{kms}^{-1}$ which yield $\Gamma \approx 0.72$ that is very close to our average value above. Therefore, this galaxy is more like a barred spiral.

3.6 NGC 4254

This galaxy is quite asymmetric and offered some important features for testing our model because of its three arms.

We began the fitting by arm B (Fig.12) and found, initially, for Γ, a value of about 0.33, very different from 0.53, the value calculated from the data taking into account the value for $v_r \approx 80\,\mathrm{kms}^{-1}$ of Ganda et al. [27] and the value for v_θ of 150 km/s from Kornreich et al. [32]. But then we found out that the reported value of $v_r \approx 80\,\mathrm{kms}^{-1}$ of Ganda et al. [27] refers to arm A in Fig. 12. Then we performed the fitting of arm A and found for Γ the average value of about 0.51 which is quite close to the observed value. **And thus, Eq. (13) is indeed a universal equation for spirals that obey Danvar equation.** Tables 6, 7 and 8 show the fittings for the different arms.

Fig. 12. The three arms of NGC 4254 considered for the fitting (Photo by Teresa O'Keefe and Jeff Lawrey/Adam. Block/NOAO/AURA/NSF).

Table 6. The fitted values for arm A of NGC 4254.

$\theta = -\Psi$	Γ
30^o	0.49
60^o	0.50
90^o	0.53
105^o	0.51

Table 7. The fitted values for arm B of NGC 4254.

$\theta = -\Psi$	Γ
60^o	0.32
90^o	0.32
120^o	0.33
150^o	0.35
180^o	0.34
210^o	0.33
240^o	0.33

Table 8. The fitted values for arm C of NGC 4254.

$\theta = -\Psi$	Γ
30^o	0.31
60^o	0.32
90^o	0.37

And now we can understand why this galaxy is so asymmetric: arm A has a larger radial velocity than arms B and C, and has Γ values of arms of barred spirals. With the above average values of Γ ($\Gamma_B \approx 0.33$, $\Gamma_C \approx 0.33$) for arms B and C we find that their radial velocities are approximately equal to 49.5 kms^{-1} which is much smaller than the radial velocity of arm A.

4. Discussion of results

4.1 Connections among shape, kinematics and evolution

We clearly see that the results are consistent and the parameter b of the Danvar equation is the ratio $\Gamma = v_r/v_\theta$ and is, thus, directly connected to the kinematics of the galaxy. This means that the shape of a spiral galaxy is directly connected to its kinematics and evolution. Young spiral galaxies are small and old spiral galaxies are large (unfolded). A spiral galaxy unfolds itself from the inside out throughout time up to the exhaustion of the mass of its nucleus. And it does not get tightly wound as a consequence of the unfolding and winding because of the radial velocity v_r. Of course, the bulge should diminish slowly with time since its mass is shed outward. The Milky Way is still shedding matter outwards and there is a lot of mass in its center yet, and so it will keep on going during quite a while, probably a couple of billion years.

The calculation of the parameter Γ for a galaxy from the shape of its arms provides important information on its kinematics and will be very useful for the study of spiral galaxies.

The above calculations and results mean that if v_θ varied too much with r spiral galaxies would not exist at all.

We immediately observe that most galaxies have not rotated much because of the following argumentation. Considering the ends of the arms of a spiral and taking the angular difference between them we obtain a certain $\Delta\theta$, and so we have the approximate relation

$$\frac{R_D \Delta\theta}{2R_D} \sim \frac{v_\theta}{v_r} = \Gamma^{-1} \tag{16}$$

and, thus, $\Delta\theta \sim 2\Gamma^{-1}$. For M51A, for example, $\Delta\theta \sim 2 \times 0.37^{-1}$ rad = 5.4 rad = 1.72 π rad. Taking a look at its photo we observe that it has barely completed a full turn. The same holds for M74, for which $\Delta\theta \sim 2 \times 0.31^{-1}$ rad = 6.45 rad = 2.05 π rad. For NGC 1300 we obtain the value of $\Delta\theta \sim 2 \times 0.5^{-1}$ rad = 4 rad = 1.27 π rad, which is very consistent.

4.2 Distinction between barred and non-barred spirals

We observe that with respect to the tangential velocity, barred spirals have larger radial velocities (expansion velocities) than non–barred spirals and that is why their spiral arms are more open, that is, less tightly wound. Analyzing

more galaxies we can establish what the minimum value of Γ is for a barred spiral.

4.3 Studies on asymmetry in spiral galaxies

The measurement of Γ by means of the spiral arms will enable us to study what is going on in asymmetric galaxies as we did above for the case of NGC 4254.

4.4 Consequences for the Milky Way

The tangential velocity of the Milky Way is about $220 km/s$ [24] and at our galactic longitude $v_r \approx 130 km/s$ [24], so that $\Gamma \approx 0.59$ and, thus, the Milky Way is probably a barred spiral. And now, knowing its Γ we can calculate the shape of its arms.

4.5 Cosmological consequences

We should readdress the time of galaxy formation because, according to the above results, galaxies were formed much earlier than what is presently considered. And since spirals were small when they were young, they probably were formed from quasars which is an old idea proposed by de Souza [34,36] and Arp [35].

4.6 Consequences for dark matter

We observe that the dynamics of the disk of a spiral galaxy is mainly dictated by v_r, v_θ and its ratio Γ, and v_θ is approximately constant in the disk because the driving forces that shed matter outward are radial forces and do no work on the perpendicular direction, and thus, cannot change v_θ. Therefore, dark matter plays no role in the dynamics of a spiral galaxy. Actually, if dark matter existed, Γ would not be approximately constant and, thus, spiral galaxies would not exist at all.

4.7 The AGN engine

The values of Γ, v_θ and v_r above presented can be of some help in the search for the nature of the AGN engine that powers spiral galaxies.

4.8 Flocculent and ring spiral galaxies

Taking into account what was presented above we can say that flocculent galaxies are probably formed when the shedding of matter is done mainly in the form of very large arcs of matter and thus the spiral structure gets messed up. In the case of ring galaxies, they should occur when the ejection of matter happens in the form of rings.

4.9 Irregular galaxies

Irregular galaxies are formed when the shedding of matter outward is done in a more chaotic way. In this case there may not exist any bipolar flow either. This means that in this case the outward flow has characteristics of an explosion.

5 Spirals which do not obey Danvar equation

This is the case of galaxies such as NGC 772. In this case the radial velocity is too high, much larger than the tangential velocity. It is a much more complex system and, of course, our simple approximation above cannot be valid.

6 Conclusion

We have presented a new model for the formation of the arms of spiral galaxies which is directly connected to their kinematics and evolution in which the Danvar equation is dynamically derived. The application of the model to spiral galaxies shows full consistency with available experimental data. We have then discussed some important consequences of the model with respect to the dynamics, evolution and the age of formation of spiral galaxies. The results reached in this paper show the lack of any need of far reaching conjectures, such as that of dark matter, to explain the interior dynamics of galaxies, since all data are fully representable via data on their formation. The absence of dark matter reached in this paper is independently confirmed by the discovery of anomalous redshift of light propagating within gaseous media without any relative motion between the source, the medium and the detector. This anomalous redshift was first theoretically predicted by Santilli in 1992 [37], then experimentally confirmed also by Santilli in 2009 [38], independently confirmed via new measurements done by G. West et al [39], and known as Santilli IsoRedShift, where the prefix "Iso" indicates the use of the novel Iso-

mathematics needed for quantitative treatments of propagation within physical media. When these measurements achieved on Earth's atmosphere are reported to the inhomogeneous and anisotropic medium in the interior of galaxies, Santilli IsoRedShift allows a quantitative representation of the anomalous redshift reported for galactic stars without any need for the conjecture of dark matter because light exits the medium inside a galaxy IsoRedShifted in a way depending on the travel within said medium, along the direction of stars to Earth. Additional independent confirmation, such as that via the demise of supersymmetries by recent experiments at CERN are contemplated for study at a future time.

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EXPLAINING HOW AND WHY THE MUON NEUTRINOS FLOW FASTER THEN THE SPEED OF LIGHT IN THE OPERA NEUTRINO EXPERIMENT

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Abstract

In this paper an attempt is made to explain how and why the muon neutrinos flow faster than the speed of light in the OPERA neutrino experiment by using the theory of quantized space and time and internal structure of elementary particles derived from a new model of mine [1]. Also this paper shows that:

1-This motion is achieved by two velocities. The first is a speed of $v = (c - \delta)$

in the time and length quanta. Then there is a velocity greater than that of light in the super dimension.

2-In these tests, muon neutrinos move with a velocity

$\frac{v - c}{c} = (2.48 \pm 0.28(stat) \pm 0.3(sys)) \times 10^{-5}$.

Only 23.98 m of this happens in the super dimension [2].

Key words: moton, neutrino, length quanta, time quanta, relativity

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Introduction:

On the basis of quantized space- time theory, neutrinos consist of some motons whose mass is less than the minimum amount for energy radiation and charge production. If the mass of the moton of the electron neutrino is considered as fundamental, then by dividing the mass ν_τ and ν_μ by it, the number of motons of each particle will be obtained. By using this number we can understand that the energy of the neutrino's moton is less than the energy of the electron's moton.

$$p_{\max.\min} = \frac{h\sqrt{1-\frac{v_{\max}^{\ 2}}{c^2}}}{l_0\sqrt{1-\frac{v^2}{c^2}}} \qquad (1) \qquad [1]$$

$$E_\nu \prec E_{\min.\max} = \frac{p_{\max}c^2}{v} \qquad (2) \qquad [1]$$

This formula gives the energy of each moton as a gauge. The energy of motons at any speed up to the maximum speed is a specific gauge. The energy of the neutrino's motons at each speed is less than this gauge, so their active mass is zero, and then their charge is zero.

This equation shows that, although the speed of neutrinos increases, their energy radiation remains zero.

$$q = \frac{\sqrt{8\pi\varepsilon_0 m_{activ}c^2 l_0}}{\sqrt[4]{1-\frac{v^2}{c^2}}}C = \frac{0}{\sqrt[4]{1-\frac{v^2}{c^2}}}C \qquad (3) \qquad [1]$$

There is another thing in equations 1, 2; that is speed limitation. (The maximum speed of an electron is equal to $V_{\max electron} = 299792407 m/s$)[1].The maximum speed of neutrinos in time and length quanta is equal to $V_{\max.\nu} = (c - \delta)$. If neutrinos get more energy, on the de Broglie assumption $\lambda = \frac{h}{mv}$, their wavelength will be less than the length quanta, but this impossible. Therefore, neutrinos cannot transfer the additional energy. They also cannot radiate the energy like the other charged particles (electrons), so the only solution is transferring this energy through the super dimension.

Until the speed is less than $V_{\max.\nu} = (c - \delta)$ the super-dimension separates the end and the beginning of the time and length quanta. But when there is additional energy which cannot be transferred by the time and length quanta gauges, then the super-dimension in addition to separating the time and length quanta, makes a distance by the type of super-dimension between the time and length quanta.

Super-dimension is one of the physics' dimensions, which is too condensed in low energies but it appears in high energies.

1-Range of relativity:

Relativity theory was based on the assumption that the maximum speed of particles is the speed of light. But the neutrinos of muons fly faster than the speed of light. At first it seems that relativity has collapsed, but it is not true. Actually the range of the rules of relativity is limited. There is something in relativity which has been ignored and that is the infinities in special relativity. The cause of these infinities is not considering the maximum speed of massive particles. The maximum speed of electron, positron, and proton… is equal to $V_{\max} = 299792407 m/s$.

The maximum speed of neutrinos in time and length quanta is $V_{\max.\nu} = (c - \delta)$.

This limit of speed saves us from the infinities in special relativity. In other words because the time and length are quantized, particles' wavelengths cannot be shorter than length quanta. So the infinities do not occur.

The neutrino follows relativity till this limit of speed and after that, because it cannot radiate its additional energy, uses the super dimension for its movement. In other words the major part of the neutrino's movement is done under relativity rules and the other part is performed through the super dimension.

In the $V_{\max,\nu} = (c - \delta)$ equation the amount of δ should be something that makes the Lorentz coefficient as following:

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = z \cdot 441002081 \qquad (4)$$

z:the number of motons of a neutrino

Up to this speed, the wavelength of the muon's neutrino is not shorter than length the quanta so the motion can be followed in the time and length quanta, and by passing this speed the motion should be performed through the super-dimension.

The threshold energy of using the super-dimension by neutrinos with 2ev mass is:

$E_1 = 0.88Gev \times z$ *z: the number of motons of a neutrino*

3-The quantized model of a neutrino:

By using the mass formula the minimum mass of an electron neutrino can be obtained.

$$\left|\overrightarrow{m_\nu}\right| = X.\frac{c^2}{\pi} \cdot \left|\overrightarrow{\Delta T_{n-1}}\right| \cdot \left|\overrightarrow{\Delta T_n}\right| \cdot Sin\delta^\circ \qquad (5) \qquad [1]$$

the mass of the electron anti neutrino ν_e is equal to:

$$\left|\overrightarrow{m_{\nu}^{+}}\right| = X \cdot \frac{c^2}{\pi} \cdot \left|\overrightarrow{\Delta T_{n+1}}\right| \left|\overrightarrow{\Delta T_n}\right| \cdot \left|Sin - \delta^{\circ}\right| \qquad (6) \quad [1]$$

On the basis of dimensional matrix model, until the speed of the neutrino is less than the $V_{\max.\nu} = (c - \delta)$ the dimensional matrix of the neutrino is:

$$\begin{pmatrix} \overrightarrow{\Delta T_1} & \overrightarrow{l_m} & \overrightarrow{m_1} \\ \overrightarrow{\Delta T_2} & \overrightarrow{l_{m-1}} & \overrightarrow{E_1} \end{pmatrix} \xrightarrow{\overrightarrow{s_{\nu 1}}} \begin{pmatrix} \overrightarrow{\Delta T_2} & \overrightarrow{l_{m-1}} & \overrightarrow{m_2} \\ \overrightarrow{\Delta T_3} & \overrightarrow{l_{m-2}} & \overrightarrow{E_2} \end{pmatrix} \xrightarrow{\overrightarrow{s_{\nu 2}}} ,\ldots \begin{pmatrix} \overrightarrow{\Delta T_{m-1}} & \overrightarrow{l_2} & \overrightarrow{m_{m-1}} \\ \overrightarrow{\Delta T_m} & \overrightarrow{l_1} & \overrightarrow{E_{m-1}} \end{pmatrix} \qquad (7) \quad [1]$$

$\begin{pmatrix} \overrightarrow{\Delta T_1} & \overrightarrow{l_m} & \overrightarrow{m_1} \\ \overrightarrow{\Delta T_2} & \overrightarrow{l_{m-1}} & \overrightarrow{E_1} \end{pmatrix}$: First dimensional matrix of a neutrino (First basic matrix)

$\begin{pmatrix} \overrightarrow{\Delta T_2} & \overrightarrow{l_{m-1}} & \overrightarrow{m_2} \\ \overrightarrow{\Delta T_3} & \overrightarrow{l_{m-2}} & \overrightarrow{E_2} \end{pmatrix}$: Second dimensional matrix of a neutrino (Second basic matrix)

$\begin{pmatrix} \overrightarrow{\Delta T_{m-1}} & \overrightarrow{l_2} & \overrightarrow{m_{m-1}} \\ \overrightarrow{\Delta T_m} & \overrightarrow{l_1} & \overrightarrow{E_{m-1}} \end{pmatrix}$: The last dimensional matrix.

$\overrightarrow{m_n}$: The mass of time-length sequences, n= (1, 2…m)

$\overrightarrow{E_n}$: The energy of time-length sequences, n= (1, 2…m)

The neutrino matrix with negative spin is given by:

$$\begin{pmatrix} \overrightarrow{\Delta T_1} & \overrightarrow{l_m} & \overrightarrow{m_1} \\ \overrightarrow{\Delta T_2} & \overrightarrow{l_{m-1}} & \overrightarrow{E_1} \end{pmatrix} \xrightarrow{\overrightarrow{s'_{\nu}}} \begin{pmatrix} \overrightarrow{\Delta T_1} & \overrightarrow{l_{m-1}} & \overrightarrow{m_2} \\ \overrightarrow{\Delta T_3} & \overrightarrow{l_{m-2}} & \overrightarrow{E_2} \end{pmatrix} \xrightarrow{\overrightarrow{s'_{\nu}}} ,\ldots \qquad (8) \quad [1]$$

The matrix of the electron anti neutrino with negative spin angular momentum

$$\begin{pmatrix} \overrightarrow{\Delta T_m} & \overrightarrow{l_m} & \overrightarrow{m_m} \\ \overrightarrow{\Delta T_{m-1}} & \overrightarrow{l_{m-1}} & \overrightarrow{E_m} \end{pmatrix} \xrightarrow{\overrightarrow{S_{\bar{\nu}\,1}}} \begin{pmatrix} \overrightarrow{\Delta T_{m-1}} & \overrightarrow{l_{m-1}} & \overrightarrow{m_{m-1}} \\ \overrightarrow{\Delta T_{m-2}} & \overrightarrow{l_{m-2}} & \overrightarrow{E_{m-1}} \end{pmatrix} \longrightarrow,\ldots \qquad (9) \quad [1]$$

The dimensional matrix of the electron neutrino with a speed of more than $V_{\max.\nu} = (c - \delta)$ is:

$$\begin{pmatrix} \overrightarrow{\Delta T_1} & \overrightarrow{l_m} & \overrightarrow{m_1} \\ \overrightarrow{\Delta T_2} & \overrightarrow{l_{m-1}} & \overrightarrow{E_1} \end{pmatrix} \xrightarrow{\overrightarrow{S_{\nu 1}}} [\Delta t_{1s} \ \ \Delta l_{1s} \ \ \Delta m_{1s} \ \ \Delta E_{1s}] \rightarrow \begin{pmatrix} \overrightarrow{\Delta T_2} & \overrightarrow{l_{m-1}} & \overrightarrow{m_2} \\ \overrightarrow{\Delta T_3} & \overrightarrow{l_{m-2}} & \overrightarrow{E_2} \end{pmatrix} \xrightarrow{\overrightarrow{S_{\nu 2}}} [\Delta t_{2s} \ \ \Delta l_{2s} \ \ \Delta m_{2s} \ \ \Delta E_{2s}] \rightarrow,\ldots$$

(10)

In this model, Δt_s represents the time during which the neutrino travels through the super-dimension. This time is equivalent in terms of time quanta.

In this model, Δl_s represents the length which is traveled by the neutrino in the super-dimension. This length is equivalent in terms of length quanta.

In this model, Δm_s represents the mass which is transferred by the neutrino in the super-dimension. This mass is equivalent in terms of the mass of the neutrino.

4-1:Dimensional matrix of the muon neutrino:

By knowing the mass of the electron neutrino and the muon neutrino, the number of motons of the muon neutrino will be obtained.

$$N_{motonμ} = \frac{M_{vμ}}{M_{ve}} = z \qquad (11)$$

By having the number of motons the dimensional matrix of ν_μ can be obtained.

Each two lines represent one moton.

The dimensional matrix of the muon neutrino with a velocity more than light speed is:

$$\nu_\mu : \begin{pmatrix} \overrightarrow{\Delta T_{1_{11}}} & \overrightarrow{l_{m_{12}}} & \overrightarrow{m_{1_{13}}} \\ \overrightarrow{\Delta T_{2_{21}}} & \overrightarrow{l_{m-1_{22}}} & \overrightarrow{E_{1_{23}}} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \overrightarrow{\Delta T_{1\cdot 2z}} & \overrightarrow{l_{m.2z}} & \overrightarrow{m_{1.2z}} \\ \overrightarrow{\Delta T_{2.2z}} & \overrightarrow{l_{m-1.2z}} & \overrightarrow{E_{1.2z}} \end{pmatrix} \xrightarrow{\overrightarrow{S_{v1}}} [\Delta t_{1s} \quad \Delta l_{1s} \quad \Delta m_{1s} \quad \Delta E_{1s}] \rightarrow \begin{pmatrix} \overrightarrow{\Delta T_{2_{11}}} & \overrightarrow{l_{m-1_{12}}} & \overrightarrow{m_{2_{13}}} \\ \overrightarrow{\Delta T_{3_{21}}} & \overrightarrow{l_{m-2_{22}}} & \overrightarrow{E_{2_{23}}} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \overrightarrow{\Delta T_{2.2z}} & \overrightarrow{l_{m-1.2z}} & \overrightarrow{m_{2.2z}} \\ \overrightarrow{\Delta T_{3.2z}} & \overrightarrow{l_{m-2.2z}} & \overrightarrow{E_{2.2z}} \end{pmatrix} \xrightarrow{\overrightarrow{S_w}} \rightarrow$$

(12)

4-2:Dimensional matrix of ν_τ :

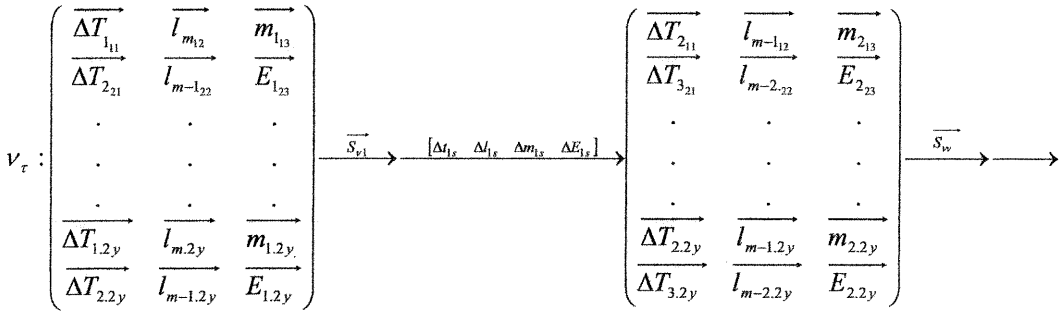
By knowing the mass of the electron neutrino and ν_τ neutrino, the number of motons of ν_τ can be obtained.

$$N_{motonτ} = \frac{M_{vτ}}{M_{ve}} = y \qquad (13)$$

By having the number of motons the dimensional matrix of ν_τ can be obtained.

Each two lines represent one moton.

The dimensional matrix of ν_τ with a velocity more than light speed is:



(14)

5-Equivalent length which is travelled by neutrinos in the super-dimension:

The equation of the speed of neutrinos in the OPERA test is:

$$\frac{v-c}{c} = (2.48 \pm 0.28(stat) \pm 0.3(sys)) \times 10^{-5} \qquad (15) \qquad [2]$$

By using this formula, and by considering the maximum speed, we can calculate the travelled path by muon neutrinos in the super-dimension.

$V_{\max_s} = 299801631 m/s$

V_s: The maximum speed in time and length quanta in addition to equivalent speed in the super-dimension

$$v = (c - \delta) \cong c \tag{16}$$

v: The maximum speed in time and length quanta

$L = 730000m$

$T_1 = 2.43494^{-3}$: Time required to route rapidly than the speed of light

$T_{\max} = 2.43502 \times 10^{-3}$: Time required to route by maximum speed in time and length quanta

$\Delta T_{\max - s} = 8 \times 10^{-8}$

$$\Delta L = V_{\max} \times \Delta T_{\max - s} = 299792458 \times 8 \times 10^{-8} = 23.98m \tag{17}$$

ΔL: Length traveled by neutrino in super-dimension

$$nl_0 = n\Delta T_0 = \frac{L - \Delta L}{l_0} = \frac{730000 - 23.98}{1.409 \times 10^{-15}} = 5.181 \times 10^{20} \tag{18}$$

$$\Delta t_s = \frac{8 \times 10^{-8}}{5.1799 \times 10^{20}} = 1.544 \times 10^{-28} s \tag{19}$$

Δt_s: The equivalent time that is added to each time quanta from super-dimension

$$\Delta l_s = \frac{23.98}{5.181\times10^{20}} = 4.63\times10^{-20}\,m \qquad (20)$$

Δl_s : The equivalent length which is added to each length quanta from super-dimension

6-Obtaining the super-dimension function:

Doing more tests and statistical analysis can help us to get suitable facts to obtain the super-dimension function. But, as a first step, we can say that the range and domain of its function is as follows:

$$F(\Delta t_s, \Delta l_s, \Delta m_s) = \Delta E_s \qquad (21)$$

$$\Delta E_s = E_\nu - \frac{E_{\nu 0}}{\sqrt{1-\frac{{\nu^2}_{\max.\nu}}{c^2}}} \qquad (22)$$

ΔE_s :Equivalent additional energy which transferred by muon neutrinos in super-dimension

Δl_s :Equivalent length in super-dimension

Δt_s : Equivalent time in super-dimension

Δm_s :Equivalent additional mass transferred in super-dimension

7-Conclusion:

1-The OPERA test shows that the time and length are quantized. To avoid the wave length being smaller than the length quanta, neutrinos use an additional dimension for their travel.

2-This test shows that, there is a limitation for the speed of elementary particles in time and length quanta. This limit of speed is protected by the relativistic charge law. Because the neutrino's particles are not charged, they cannot radiate their additional energy on the basis of charge formula, so to transfer their additional energy, they use a super-dimension. Besides this is another proof for the limit of speed and relativistic charge law in the time and length quanta.

3-This test shows there is a super-dimension; otherwise relativity will collapse. Relativity is true for the particles which are equal or heavier than the electron up to the speed v=299792407m/s. Also it is true for neutrinos up to the velocity $V_{\max,\nu} = (c - \delta)$.

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COMMENT ON "RELATIVITY THE HOLY GRAIL OF PHYSICS?"

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Abstract

In a recent article, 'Relativity the Holy Grail of Physics?', which appeared in the Hadronic Journal (Hadronic Journal, **34** 125), Spoljaric proposed some interesting new insights into the relationships between classical, quantum and relativistic mechanics. While the thoughts enunciated should be considered seriously, some of the manipulations are shown to require further examination and possibly correction. Also, some of the assertions made are seen to need elaboration and explanation in order to justify them fully.

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1. Introduction.

In a recent edition of the Hadronic Journal [1], Spoljaric introduced several interesting new ideas and drew some novel conclusions on the basis of these. However, some of the manipulations accompanying all this do give some cause for concern. In addition to this, some claims made seemed to carry little or no clearly identifiable justification and this seems unfortunate since he may well have had some very good points to make.

2. Manipulations.

His starting point for the entire discussion was several of the usual equations of relativistic mechanics, beginning with

$$M = m_0/\sqrt{1-(v/c)^2}$$

where, as usual, M is the relativistic mass, v the speed, c the speed of light, m_0 the rest mass.
This equation is then used to write the energy $E = mc^2$ and the relativistic momentum $\boldsymbol{p} = M\boldsymbol{v}$ in terms of the rest mass:

$$E = m_0c^2/\sqrt{1-(v/c)^2} \quad \text{and} \quad \boldsymbol{p} = m_0\boldsymbol{v}/\sqrt{1-(v/c)^2} \qquad \text{(i)}$$

These latter two equations are then combined, again as usual, to give

$$E = \sqrt{(m_0\,c^2)^2 + (pc)^2}$$

He then points out that

$$E = hf$$

and so it follows that

$$hf = \sqrt{(m_0c^2)^2 + (pc)^2}.$$

Surprisingly, this identification seems new in relativistic mechanics and yet it does introduce Planck's constant into things and this is something normally taken to herald quantum mechanical ideas. From this latter equation, it follows that, if $m_0 = 0$, since $f = c/\lambda$,

$$\lambda = {}^{h}/_{p}$$

but if $p = 0$

$$\lambda = {}^{h}/_{m_0 c} \qquad \text{(ii)}$$

Then, using the equation for M in terms of m_0 in combination with this latter result leads to

$$E = hc^2/\lambda\sqrt{(c^2 - v^2)} \qquad \text{(iii)}$$

However, it is important to remember that this result only holds only for the case when $\boldsymbol{p}$ is zero and, referring to the second equation in (i), it is seen that $p = 0$ implies that either $m_0 = 0$ or $v = 0$. Since $m_0 = 0$ would create problems with (ii), it must be assumed that $v = 0$ and, if that is inserted in (iii), the result is that that equation becomes

$$E = hc/\lambda = hf,$$

which simply brings one back to the starting point.

This simple extension to the analysis illustrates how careful one must be with mathematical manipulations not to forget what has been assumed earlier in the work. Also, this example must mean that all other manipulations require careful checking.

3. Brief Discussion of Claims Made.

At the end of section 2 he claims that 'all that remains of Maxwell's (wave)Equations is the constant c' but, since Maxwell's equations do not seem to be mentioned before this, it is difficult to see how this claim can be justified on the basis of the content of the article. The section goes on to claim, again without any apparent justification, that 'as the Light necessarily incorporates $\hbar$, every arbitrary mass-based formalism of non-relativistic Quantum Mechanics could have been avoided, and as such is fictitious'. This is a staggering claim to make, especially the very last piece, and surely requires a great deal of further amplification and justification. The same must be true also of the claim that 'The Light renders Einstein's mass-based theory of General Relativity premature'.

The claim at the end of section 3 is no less amazing in that it is said that 'the absence of Newton's Second Law means there are no Newtonian Laws of Motion'. This is an extremely difficult notion to accept, especially considering the lack of careful argument in justification for it. The same may be said of the remainder of the section and, indeed, for the content of the short section 4. It might be noted that, in all cases, a detailed explanation of what is meant precisely by 'The Light' would be helpful since it appears to lie at the heart of virtually everything but is not explained lucidly.

Entropy is, of course, an extremely complicated concept which is probably truly understood by no-one. However, introducing it via the Bekenstein-Hawking expression for the entropy of a black hole is not an auspicious beginning for any discussion of that topic. However, it is the final conclusion that, if the formula holds, 'then we have a consistent (non-classical) relativistic unification of both the constants *G, c, k* and entropy'. This, combined with the final claim that 'all that remains of Classical Physics are the constants *G, c, k* and the concept of entropy', are staggering notions which, if true, would be bound to really revolutionise physics as we know it.

The claims are made. It would be good to see a detailed justification for them.

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ROLE OF THE FINE STRUCTURE RATIO IN FITTING THE SEMF ENERGY CONSTANTS AND SUPER HEAVY MAGIC NUMBERS

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Abstract

In the semi empirical mass formula, the ratio of Coulombic and pairing energy constants is close to the ratio of the fine structure constant and the strong coupling constant. In this paper considering α and α_s, an attempt is made to understand the origin of the SEMF energy constants. Finally observed magic numbers are fitted and new heavy magic numbers are predicted. The two super heavy magic series are 138,150,160,178,184,200,... and 136,126,108,98,82,80,... With these heavy magic neutron numbers many stable super heavy elements can be predicted.

Keywords: Fine structure ratio; Strong coupling constant; Proton number; Nucleon mass number; Semi empirical mass formula; Magic numbers;

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1 Introduction

One of the first nuclear models, proposed in 1935 by Bohr, is based on the short range of nuclear forces, together with the additivity of volumes and of binding energies. It is called the liquid-drop model. Nucleons interact strongly with their nearest neighbours, just as molecules do in the drop of a liquid. Therefore, one can attempt to describe their properties by the corresponding quantities,viz., the radius, the density, the surface tension and the volume energy. An excellent parametrization of the binding energies of nuclei in their ground state was proposed in 1935 by Bethe and Weizsacker. This formula relies on the liquid-drop analogy but also incorporates two quantum ingredients. One is an asymmetry energy which tends to favour equal numbers of protons and neutrons. The other one is a pairing energy which favours configurations where two identical fermions are paired. In this formula, the coefficients are chosen so as to give a good approximation to the observed binding energies. Nuclear experiments indicate that atomic nuclei possess a shell-structure also.

The major task is to study the proton-neutron stability, nuclear binding energy and magic numbers in a unified way. Considering the fine structure ratio and the strong coupling constant, an attempt is made to understand the 'origin' of the semi empirical mass formula energy constants. It is noticed that, fine structure ratio plays an interesting role in the generation of magic numbers.

2 To implement the fine structure ratio in nuclear structure

2.1 To fit the proton-nucleon stability

It is well established that, in β decay, neutron emits an electron and transforms to proton. Thus the nuclear charge changes and the nucleus gets stability. From the semi empirical mass formula ([1],[2]) it is established that,

$$Z \cong \frac{A}{2 + (E_c/2E_a)\, A^{2/3}}. \qquad (1)$$

where Z = number of protons of the stable nucleus and A =number of nucleons in the stable nucleus. E_a and E_c are the asymmetry and coulombic energy constants. It is noticed that

1. For most of the stable elements in between $Z \approx 10$ to 60, upper limit of the stable mass number is very close to

$$A_S \cong 2Z + \left(\alpha \cdot Z^2\right) \tag{2}$$

where α is the fine structure ratio.
If Z= 32, $A_S = 71.47$; Z= 39, $A_S = 89.10$; Z= 42, $A_S = 96.87$; Z=50, $A_S = 118.24$; Z=53, $A_S = 126.50$ and Z=60, A_S =146.27.

2. Let us define a new number in the following way:

$$\beta \cong \frac{1}{\sqrt{\alpha}} \cdot \frac{m_e}{m_n} \cong \frac{1}{157.069} \cong 6.36663 \times 10^{-3} \tag{3}$$

where m_n is the rest mass of neutron and m_e is the rest mass of electron.

3. For light and heavy elements, stable mass number is very close to

$$A_S \cong 2Z + \beta Z^2 \tag{4}$$

If Z= 21, $A_S = 44.8$; Z= 29, $A_S = 63.35$; Z=47, $A_S = 108.06$; Z=79, $A_S = 197.73$ and Z=92, $A_S = 237.88$. This idea can be given a chance in estimating the stable super heavy elements.

4. Considering A as the fundamental input its stable Z = Z_S takes the following form:

$$Z_S \cong \frac{1}{\beta}\left(\sqrt{\beta A + 1} - 1\right) \tag{5}$$

Thus Green's stability formula in terms of Z takes the following form.

$$\frac{0.4A^2}{A + 200} \cong A_S - 2Z \approx \beta Z^2. \tag{6}$$

5. Another interesting observation is that

$$(m_n - m_p)\, c^2 \cong \ln\left(\frac{1}{\sqrt{\beta}}\right) m_e c^2 \cong 1.29198 \text{ MeV} \tag{7}$$

Here m_n, m_p and m_e are the rest masses of neutron, proton and electron respectively.

2.2 Strong coupling constant and the characteristic strong interaction potential

Characteristic QCD energy scale is close to 200 MeV. Its half energy is close to 100 MeV. The strong coupling constant α_s is a fundamental parameter of the Standard Model. It plays a more central role in the QCD analysis of parton densities in the moment space. QCD does not predict the actual value of α_s, however, it definitely predicts the functional form of energy dependence α_s. The value of α_s, at a given energy or momentum transfer scale, must be obtained from experiment. Determining α_s at a specific energy scale is therefore a fundamental measurement, to be compared with measurements of the electromagnetic coupling α, of the elementary electric charge e or of the gravitational constant G. Considering perturbative QCD calculations from threshold corrections, its recent obtained value [3] at N^3LO is $\alpha_s \cong 0.1139 \pm 0.0020$. At lower side $\alpha_s \cong 0.1139 - 0.002 = 0.1119$ and at higher side $\alpha_s \cong 0.1139 + 0.002 = 0.1159$.

Assumption: The characteristic strong interaction potential is

$$P_s \cong \frac{\hbar^2}{2m_p\left(\frac{R_0}{2}\right)^2} + \frac{\hbar^2}{2m_n\left(\frac{R_0}{2}\right)^2} \cong \alpha_s \cdot \frac{m_p c^2 + m_n c^2}{2} \tag{8}$$

where $R_0 \cong 1.25$ to 1.26 fm. Giving a primary significance to the proton rest mass and considering R_0 as 1.255 fm,

$$P_s \cong \frac{\hbar^2}{m_p\left(\frac{R_0}{2}\right)^2} \cong \alpha_s \cdot m_p c^2 \cong 105.3942 \text{ MeV} \tag{9}$$

Thus $\alpha_s \cong \frac{P_s}{m_p c^2} \cong 0.112327961$. On the cosmological scale, this characteristic potential plays a very crucial and interesting role in fitting the reduced Planck's constant and the proton-electron mass ratio. Please see the appendix.

2.3 To understand the origin of the SEMF energy constants

It can be suggested that nuclear binding energy is a combined phenomenon of strong and electromagnetic interactions. Even though the strong coupling constant is proposed for understanding the mystery of the strong interaction till today it was not implemented in understanding the nuclear binding energy. Here the proposed procedure is not inline with the current methods of fitting the SEMF binding energy constants. But this procedure includes both the fine structure ratio and the strong coupling constant.

Let pairing energy constant $= E_p$, asymmetry energy constant $= E_a$, surface energy constant $= E_s$, volume energy constant $= E_v$ and coulombic energy constant $= E_c$.

2.3.1 Coulombic, pairing and asymmetric energy constants

Considering the coulombic repulsion, coulombic energy constant is close to

$$E_c \cong \alpha \cdot P_s \cong 0.7691 \text{ MeV} \tag{10}$$

Considering the strong attraction, pairing energy constant is close to

$$E_p \cong \alpha_s \cdot P_s \cong 11.839 \text{ MeV} \tag{11}$$

Asymmetric energy constant is close to

$$E_a \cong 2E_p \cong 23.677 \text{ MeV} \tag{12}$$

The important observations are

$$\frac{E_c}{E_p} \cong \frac{\alpha}{\alpha_s} \tag{13}$$

$$\frac{E_c}{E_a} \cong \frac{E_c}{2E_p} \cong \frac{\alpha}{2\alpha_s} \tag{14}$$

2.3.2 Volume and surface energy constants

Considering the coulombic repulsion,

$$\epsilon_c \cong \sqrt{P_s E_c} \cong 9.105 \text{ MeV} \tag{15}$$

Considering the strong attraction,

$$\epsilon_s \cong \sqrt{P_s E_p} \cong 35.323 \text{ MeV} \tag{16}$$

Now

$$\sqrt{\epsilon_c \epsilon_s} \cong (\alpha \alpha_s)^{\frac{1}{4}} P_s \cong 17.833 \text{ MeV} \tag{17}$$

It is noticed that, sum of volume and surface energy constants is close to

$$E_s + E_v \cong 2\sqrt{\epsilon_c \epsilon_s} \cong 35.666 \text{ MeV} \tag{18}$$

$$(E_s, E_v) \cong \sqrt{\epsilon_c \epsilon_s} \pm 2E_c \cong (19.371, 16.295) \text{ MeV} \tag{19}$$

2.3.3 Relation between E_v, E_s, E_a and E_p energy constants

It is also noticed that, sum of asymmetry and pairing energy constants is close to

$$E_a + E_p \cong 2\sqrt{\epsilon_c \epsilon_s} \cong 35.666 \text{ MeV} \tag{20}$$

Thus it seems reasonable to express all the four energy constants in one relation as

$$E_s + E_v \cong E_a + E_p \cong 2\sqrt{\epsilon_c \epsilon_s} \cong 35.666 \text{ MeV} \tag{21}$$

Thus, $E_a \cong 2\sqrt{\epsilon_c \epsilon_s} - E_p \cong 23.82774$ MeV.

2.3.4 The semi empirical mass formula and the nuclear stability factor

The semi empirical mass formula is

$$BE \cong AE_v - A^{\frac{2}{3}} E_s - \frac{Z(Z-1)}{A^{\frac{1}{3}}} E_c - \frac{(A-2Z)^2}{A} E_a \pm \frac{1}{\sqrt{A}} E_p \tag{22}$$

Z	A	obtained BE in MeV	measured BE in MeV	%Error
26	56	493.13	492.254	-0.178
28	62	547.69	545.259	-0.446
34	84	729.33	727.341	-0.273
50	118	1008.69	1004.950	-0.372
60	142	1185.07	1185.145	0.0065
79	197	1556.98	1559.40	0.155
82	208	1627.59	1636.44	0.54
92	238	1805.92	1801.693	0.235

Table 1: **SEMF binding energy with the proposed energy constants**

Here the important point to be noted is that, energy constants are having strong inter-relation with the coupling constants and are not arbitrary. With these energy constants qualitatively and quantitatively nuclear binding energy can be fitted [1]. For calculation purpose $E_a \cong 2E_p \cong 23.677$ MeV is considered.

Note that authors are trying to co-relate the SEMF energy constants with α, α_s and $\mathrm{m_p}$. It is a new kind of estimation of the SEMF binding energy constants. In table-1 considering the magic (proton and neutron) numbers, qualitatively within the range of $(Z = 26; A = 56)$ to $(Z = 92; A = 238)$ nuclear binding energy is calculated and compared with the measured binding energy [4]. If this procedure is found to be true and valid then with a suitable fitting procedure qualitatively and quantitatively magnitudes of the proposed binding energy constants can be refined. Authors are working in this new direction. The two interesting observations are:

$$\frac{1}{\alpha} \approx \frac{E_s}{E_c}\sqrt{\frac{E_a}{E_c}} \cong 139.75 \tag{23}$$

$$\frac{1}{\beta} \approx \frac{E_a}{E_c}\sqrt{\frac{E_s}{E_c}} \cong 154.50 \tag{24}$$

Considering these two numbers as the two characteristic limits, proton-

nucleon stability can be expressed as

$$A_S \cong 2Z + (\alpha \text{ or } \beta)\, Z^2 \tag{25}$$

where β can be called as the proton-nucleon stability number. α and β can be co-related as

$$\frac{\beta}{\alpha} \cong 1 - \sqrt{\frac{E_c}{2E_a}} \cong 1 - \sqrt{\frac{\alpha}{4\alpha_s}} \tag{26}$$

2.4 To fit and predict the magic numbers

After 1932, experimental data on nuclei repeatedly revealed that in the nucleus, there exists a series of magic numbers, called so, because nuclei with these values for the atomic number Z and/or neutron number N, showed exceptional stability. The observed magic numbers are: 2,8,20,28,50,82 and 126. The separation energy for one proton or one neutron is very large for a nucleus with Z or N magic. The probability of nuclei with neutron numbers 20,28,50,82 or 126 capturing a neutron is much less than that of their neighbouring nuclei.

1. Considering α as the characteristic stability number, in a reverse order, magic numbers 82,50,28,20 and 8 can be fitted ([5]-[10]).

2. Data indicates that for heavy elements stability is achieved when neutron-proton ratio approaches ≈ 1.6. Let us define a new number k in the following way:

$$k \cong \left(\frac{m_n - m_p}{2m_e}\right)^2 - 1 \cong 0.6014 \tag{27}$$

3. Magic numbers can be fitted with the following empirical relation.

$$M_n \cong \left[k^n \cdot \left(\frac{1}{\alpha}\right)\right] + [(-1)^n \cdot (n \cdot k)] \tag{28}$$

where $n \cong 1, 2, 3, 4, 5$ and 6. Thus $M_n \cong$ 81.81,50.77,28.0,20.33,7.77 and 10.09. After rounding off the values to the nearest even integers, we get, 82,50,28,20,8 and 10. At n=7, $M_n \cong -0.31$ and thus the

series seems to be closed at n=6 itself. Here magic numbers 2 and 126 are missing. The predicted new magic number is 10. The proposed expression is very simple. It clearly indicates that - α is strongly interconnected with nuclear closed shells.

4. Heavy magic numbers above and below 126 can be predicted with the following expression.

$$M_n \approx \left[1 \pm \alpha^{\frac{1}{n}}\right] \cdot \left(\frac{1}{\alpha}\right) \tag{29}$$

For, $\left(1 + \alpha^{\frac{1}{n}}\right)$ series, $M_n \approx$ 138.04,148.74,163.62,177.09,188.26,197.38,...
For, $\left[1 - \alpha^{\frac{1}{n}}\right]$ series, $M_n \approx$ 136.04,125.33,110.46,96.98,85.81,76.68,...
Fine tuning can be achieved if it is assumed that

$$M_n \cong \left[\left(1 \pm \alpha^{\frac{1}{n}}\right) \cdot \left(\frac{1}{\alpha}\right)\right] + [(-1)^n \cdot (n \cdot k)] \tag{30}$$

For, $\left[1 + \alpha^{\frac{1}{n}}\right]$ series, $M_n \cong$ 137.43,149.94,161.18,179.49,185.25,200.99,...
For, $\left[1 - \alpha^{\frac{1}{n}}\right]$ series, $M_n \cong$ 135.43,126.53,108.65,99.39,82.80,80.29,...
After rounding off the values to the nearest even integers, we get,
$M_n \cong$ 138,150,160,178,184,200,... and 136,126,108,98,82,80,...
Magic number 184 can be observed in $\left[1 + \alpha^{\frac{1}{n}}\right]$ series and 126 can be observed in $\left[1 - \alpha^{\frac{1}{n}}\right]$ series.

3 Discussion

Proposed semi empirical relations clearly indicate the role of the fine structure ratio and the strong coupling constant in fitting the nuclear stability, nuclear binding energy constants and the magic numbers.

4 Conclusion

Qualitatively and quantitatively proposed semi empirical relations can be given a chance in understanding and developing the unified concepts in nuclear shell structure.

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Appendix: Cosmological atomic coincidences

There are 4 interesting and accurate relations. But their interpretation seems to be very sensitive and negative to the modern cosmological observations and quantum mechanical interpretations.

Observation-1: The reduced Planck's constant

It is noticed that

$$\sqrt{\exp\left(\frac{\mathrm{P_s}}{\mathrm{m_e c^2}}\right)} \cdot \frac{G m_e^2}{c} \cong \hbar \tag{31}$$

where G is the gravitational constant and m_e is the electron rest mass. It may be an accidental coincidence.

Observation-2: Proton-Electron mass ratio

It is also noticed that

$$\sqrt{\exp\left(\frac{\mathrm{P_s}}{\mathrm{m_e c^2}}\right)} \cdot \sqrt{\frac{2 G m_e H_0}{c^3}} \cong \frac{m_p}{m_e} \tag{32}$$

where H_0 is the present Hubble's constant $\cong$ 70 to 71 Km/sec/Mpc. It may also be an accidental coincidence.

Observation-3: Combination of observation1 and observation2

Let the cosmic closure density is,

$$\rho_0 \cong \frac{3 H_0^2}{8 \pi G} \tag{33}$$

Volume of the universe in a Euclidean sphere of radius $\left(\frac{c}{H_0}\right)$ is equal to

$$v_0 \cong \frac{4\pi}{3}\left(\frac{c}{H_0}\right)^3 \tag{34}$$

Mass of the universe in a Euclidean sphere is

$$M_0 \cong \rho_0 \cdot v_0 \cong \frac{c^3}{2GH_0} \tag{35}$$

Thus considering relations (31 to 35) a very strange relation is generated.

$$\hbar \cong \frac{Gm_p\sqrt{m_e M_0}}{c} \tag{36}$$

Thus one can relate the cosmological and atomic physical constants in the following way.

$$H_0 \cong \frac{Gm_p^2 m_e c}{2\hbar^2} \cong 70.74955 \text{ Km/sec/Mpc} \tag{37}$$

Bohr radius of hydrogen atom can be expressed as

$$a_0 \cong \frac{4\pi\varepsilon_0 Gm_p M_0}{e^2} \cdot \frac{Gm_p}{c^2} \tag{38}$$